

Finite Element Verification of Saint-Venant Principle based on Abaqus

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Abstract

The Saint-Venant principle is a classical theory in solid mechanics. The core idea is that the influence of external force on the internal stress and deformation of the solid will gradually weaken to be negligible away from the load area. In this paper, Abaqus finite element software is used to establish a three-dimensional model to study the influence of different load distribution (point load, line load and uniform load) on the stress field. The cube model is selected as the analysis object, and the concentrated load and uniform distribution load are applied. By analyzing the stress distribution far away from the load area, it is observed whether it conforms to the conclusion of Saint-Venant principle. The results show that the distribution of stress and deformation tends to be stable when far away from the load area, which is independent of the load form, and further supports the theoretical basis and engineering application value of the Saint-Venant principle.

Keywords

Saint-Venant Principle; Finite Element Analysis; Abaqus; Stress Field Distribution; Lastic Mechanics.

1. Introduction

The Saint-Venant principle is an important basic principle in elastic mechanics. The core idea of the Saint-Venant principle is that the influence of external force on stress and deformation in the elastomer will be significantly weakened at a distance from the action area, and has nothing to do with the specific distribution of the load. It is only related to its total effect. This principle provides a theoretical basis for simplifying the structural analysis of complex problems, so it is of great significance in engineering design, numerical simulation and material research.

Teng [1] established concrete column and I-shaped steel beam models by ANSYS and ABAQUS respectively, and applied concentrated force and equivalent distributed force for finite element simulation. The results show that the load only significantly affects the near-end stress, and the far-end stress distribution is basically the same, which verifies the effectiveness of the Saint-Venant principle. Zhang Xiaofeng [2] pointed out that the Saint-Venant 's equation, which is derived from the integration of the three-dimensional N-S equation, is embedded in the boundary information of the river bed and loses its universality. The empirical formula with undetermined coefficients is an inevitable choice to maintain its universality. This method has important scientific demonstration significance for engineering problems such as sediment carrying capacity of water flow. Wang Qing [3] used ANSYS to apply equivalent concentrated force and uniform load to carry out finite element simulation on a rectangular plate with a circular hole. The results show that the load only significantly affects the stress near the loading end, and the stress distribution away from the end area is basically the same. The principle of Saint-Venant is verified numerically, and the teaching application method of the principle is explored. Saiyan and Paushkin [4] carried out numerical simulation of I-beam based on LIRA software, and studied the law of shear stress disturbance under different kinematic boundary

conditions. The results show that the stress disturbance attenuates rapidly away from the boundary area, and the extended form of Saint-Venant principle suitable for kinematic boundary conditions is extended and verified. Kutlu Darılmaz et al. [5] used the hybrid finite element method to carry out Saint-Venant torsion analysis on the cross-sections of orthotropic composite materials and functionally graded materials with arbitrary shapes, which provides an efficient and accurate numerical method for the torsion calculation of heterogeneous components with complex cross-sections. Chen Li Zichen [6] and others proposed a general analytical solution strategy under polynomial load for the problem of plane straight beam in elastic mechanics. Combined with the Saint-Venant principle, the main boundary conditions were accurately satisfied, and the correctness and applicability of the method were verified by finite element comparison. Inspired by the fish fin structure, Yokota and Barthelat [7] proposed the design of superelastic cellular configuration beam, which breaks through the limitation of Saint-Venant's principle by adjusting the axial, bending and shear stiffness, and realizes large-scale continuous deformation under local driving. The effectiveness of the deformation mechanism and design criteria is verified by numerical simulation and 3D printing experiments. Song Shaoyun et al. [8] proposed the Saint-Venant principle in finite element meshing, and pointed out that the mesh density in local area had little effect on the calculation accuracy at a distance. Based on this, a grid division method of global rough division + local refinement of key areas is established. The example shows that this method can significantly improve the efficiency while ensuring the calculation accuracy, and provide an efficient and practical scheme for finite element pre-processing.

In practical engineering, many structural problems need to be analyzed by numerical methods. In particular, the finite element method (FEM) has become the preferred tool for studying the stress distribution of elastic bodies due to its flexibility and efficiency. Finite element analysis can not only accurately simulate complex geometric shapes and load conditions, but also provide a new perspective for verifying classical theories. By verifying the Saint-Venant principle through finite element simulation, we can more intuitively understand its scope of application and limitations, so as to provide a basis for simplified assumptions in engineering applications.

In this paper, Abaqus finite element analysis software is used to design a beam model. By changing the loading mode (point load, line load and uniform load), the distribution law of stress field near the load area and far field is compared and analyzed, so as to verify the effectiveness of Saint-Venant principle. The research results are helpful to deepen the understanding of classical mechanics theory, and provide theoretical support for the rationality of load simplification hypothesis in practical engineering.

2. Problem and Model Description

2.1. Problem Description

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2.2. Geometric Model

The model is cuboid, $L = 1000\text{mm}$, $W = 200\text{mm}$, $H = 300\text{mm}$. One end of the cube is applied with a uniform load of 40 N equivalent to 1000 N.

2.3. Material Parameter

It is assumed that the cuboid material is isotropic linear elastic material with elastic modulus $E = 200\text{ GPa}$.

Elastic modulus : $E = 200\text{ GPa}$

Poisson 's ratio : $\nu = 0.3$

2.4. Mesh Subdivision

The model is divided into three-dimensional solid grids, and 8-node linear brick element (C3D8R) is selected. The mesh size is selected as 10 mm to ensure the balance between calculation accuracy and efficiency. See Fig. 1.

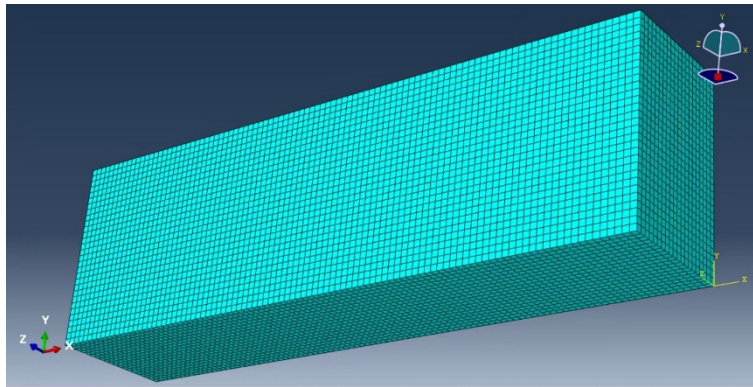


Fig 1. Grid division diagram

2.5. Boundary Conditions and Loads

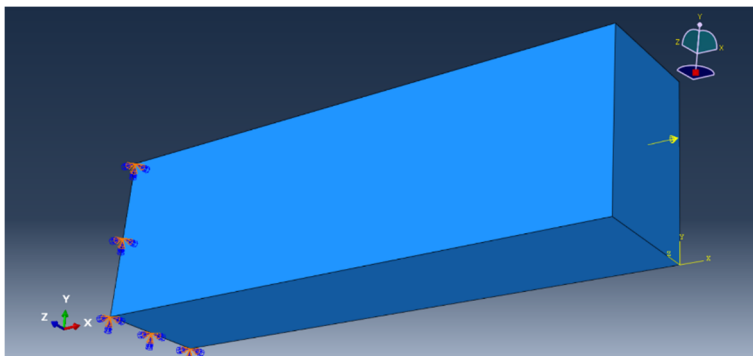


Fig 2. Concentrated load diagram

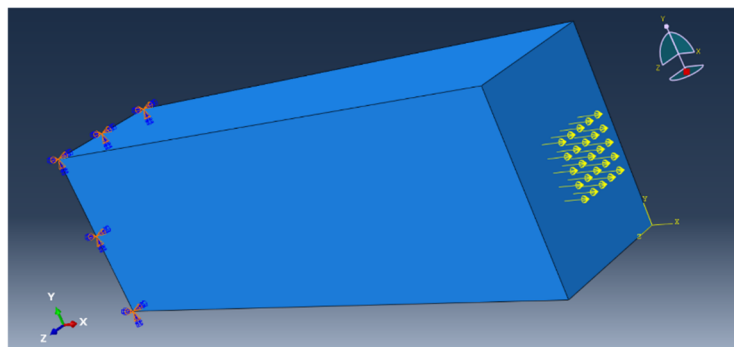


Fig 3. Uniform load diagram

Boundary conditions : The left end of the cube ($x = 0$ plane) is completely fixed, and all degrees of freedom (displacement and rotation) are constrained.

Load conditions :

1. Concentrated load : an inward concentrated load of 1000 N is applied at the center of the right end of the cube ($x = 1$ plane).
2. Uniformly distributed load : A uniformly distributed normal load is applied to the right end of the cube, with a total size of 40N.

2.6. Analytical Procedure

1. Using static analysis steps to calculate the stress field distribution.
2. The stress values on different sections along the length direction (x direction) in the model are extracted, especially the changes near the load surface and away from the load area.

3. Results and Some Discussion

3.1. Stress Distribution Results

Near the load area: Different load forms lead to significantly different stress concentration phenomena. For example, the stress concentration caused by point load is the largest, while the uniform load is the smallest.

Away from the load area : As the distance increases, the stress distribution under each load form tends to be consistent, which verifies the applicability of the Saint-Venant principle.

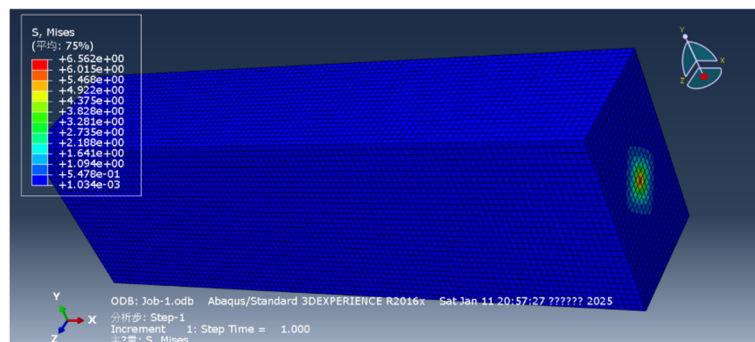


Fig 4. Stress distribution result diagram

Uniformly distributed load : The stress distribution of uniform load is relatively stable in the area near the load surface. In the region far away from the loading surface, similar to the result of concentrated load, the stress field tends to be consistent.

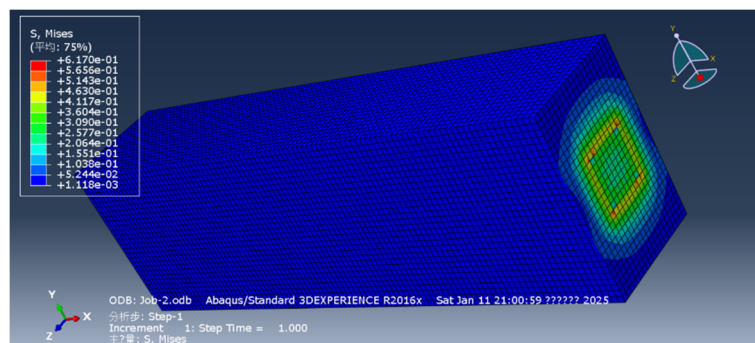


Fig 5. The result of uniform distribution load

3.2. Stress Attenuation Analysis

A series of feature points are selected along the load direction in the cuboid model, and the stress values of each point under different working conditions are extracted. The distance from the loading surface is the abscissa, and the stress value is the ordinate to draw the curve. It can be seen from the curve that under the condition of uniform load, the stress is basically stable in the process of far away from the loading surface, with slight fluctuation but little change. Under the concentrated load condition, the stress decreases sharply near the loading point. After the distance from the loading point is about 3-5 times the width of the cuboid, the stress value tends to be stable and is similar to the stress value at the corresponding position under the uniform load condition. This shows that after satisfying a certain distance condition, the influence of the concentrated load and the static equivalent uniform load on the stress distribution at the far end of the structure tends to be consistent, which verifies the Saint-Venant principle.

3.3. Discussion and Analysis

The numerical results show that the load form has little effect on the stress distribution in the far field region.

The Saint-Venant principle is verified: the far-field response is only related to the total load and has nothing to do with the details of the load distribution.

4. Conclusion

In this study, with the help of Abaqus software, the Saint-Venant principle was successfully verified by establishing a three-dimensional cuboid model and applying uniform load and concentrated load. The research shows that after a certain distance from the load area, the influence of the change of local load form on the stress distribution of the structure gradually decreases and tends to be stable. This provides a theoretical basis and simplified calculation method for engineering designers to deal with complex load problems, which is helpful to improve the efficiency and safety of engineering design. Future research can be further extended to the verification and application of Saint-Venant 's principle under complex structural forms, different material properties and dynamic loads.

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