ISSN: 2688-8645

Kinematics Analysis of 3-RCU 3D Model Parallel Robot

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Abstract

This study focuses on the 3-RCU parallel robot as the support mechanism. First, the configuration design of the 3-RCU parallel mechanism is completed, and its degree of freedom (3, including z-axis translation and rotation around x and y axes) is calculated using the spatial mechanism degree of freedom formula, followed by the construction of the mechanism's kinematic model. Through accurate dynamic modeling, the research captures the nonlinear variation of support interface stiffness, providing key parameters for the subsequent design of parallel mechanisms and laying the theoretical foundation for constructing time-varying dynamic models.

Keywords

3-RCU 3D Model; Position and Orientation Analysis; Parallel Robot.

1. Introduction

Compared to traditional serial mechanisms, the 3-RCU parallel mechanism achieves three degrees of freedom—z-axis translation and rotation about the x- and y-axes—through the symmetrical arrangement of a moving platform, a stationary platform, and three RCU linkages[1-2]. Its multi-closed-loop structural characteristics significantly outperform conventional support mechanisms in both load-bearing capacity and motion precision, establishing it as a key technological solution for addressing the challenges of machining low-stiffness workpieces in high-end manufacturing[3-4].

2. Structural Analysis of 3-RCU 3D Model

The 3-RCU parallel mechanism comprises one moving platform, one fixed platform, three minor linkages, and three major linkages, totalling eight components. It incorporates three rotary pairs (R), three cylindrical pairs (C), and three universal joints (U), totalling nine kinematic pairs. The rotary pairs (R) possess one degree of freedom, the cylindrical pairs (C) possess two degrees of freedom, and the universal joints (U) possess two degrees of freedom. The mechanism exhibits no virtual constraints and no local degrees of freedom. Applying the formula for calculating degrees of freedom in spatial mechanisms, the formula for the 3-RCU parallel mechanism can be expressed as:

$$M = d(n-g-1)+\sum_{i=1}^{g} f_i + v + \varphi \tag{1}$$

Where M denotes the number of degrees of freedom in the mechanism, d represents the order of the parallel mechanism, n signifies the total number of components in the parallel mechanism, g indicates the total number of kinematic pairs in the parallel mechanism, fi denotes the number of degrees of freedom for the i-th kinematic pair in the parallel mechanism, v signifies the number of geometric constraints in the parallel mechanism, and φ represents the number of local degrees of freedom for the object under study.

Substituting the mechanism parameters yields the number of degrees of freedom for the parallel mechanism.

The mirror-milling support mechanism for thin-walled components is a 3-degree-of -freedom parallel mechanism, with its motion diagram shown in Figure 2. This mechanism comprises a top-mounted moving platform $B_1B_2B_3$, A bottom stationary platform $A_1A_2A_3$ Comprising three identical and symmetrically distributed side chains $A_iB_i(i=1,2,3)$. Each branch $A_iB_i(i=1,2,3)$ comprises a rotary pair R, a cylindrical pair C, and a universal joint U. Where $A_i(i=1,2,3)$ denotes the axis of the rotating pair R. $B_i(i=1,2,3)$ The universal joint centre is designated as U. A fixed coordinate system and a relative coordinate system are established on the stationary platform and moving platform respectively. The centres of the stationary and moving platforms are taken as origins O and O' to establish coordinate systems Oxyz and O'x'y'z' respectively, with their corresponding axis directions maintained consistent. The three-dimensional model created using Solidworks is shown in Figure 1.

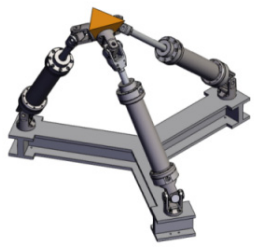

Figure 1. 3-RCU 3D model

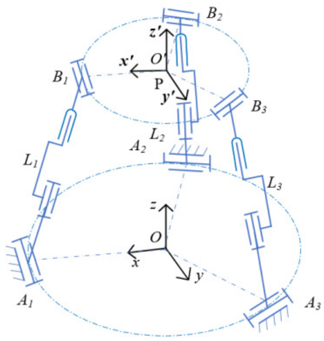

Figure 2. 3-RCU kinematic diagram.

3. Position and Orientation Analysis

Structural analysis of the parallel mechanism reveals its three degrees of freedom at the end effector (with the line connecting the centres of the moving and stationary platforms as the central axis): translation along the z-axis, and rotation about the x-axis and y-axis. Employing analytical methods to solve the inverse kinematics of the 3-RCU mechanism, the structural parameters established in the kinematic model of Figure 2 yield the coordinates of each point within different reference frames. The coordinates of point A on the fixed platform within the fixed coordinate system are as follows:

$$A_i^0 = R\begin{pmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ 0 & 0 & 0 \end{pmatrix} \tag{2}$$

In the formula, R represents the radius of the circumscribed circle of the static platform;

$$B_i^{o'} = r\begin{pmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ 0 & 0 & 0 \end{pmatrix} \tag{3}$$

Where r denotes the radius of the circumscribed circle of the moving platform. Let $P_A(x_p, y_p, z_p)$ denote the coordinate representation of the position vector of the moving platform's origin P relative to the fixed coordinate system $O-XYZ$ .and R_B^A denote the coordinate rotation matrix of the moving platform relative to the stationary platform A. Then:

$$B_i^A = P_A + R_B^A B_i^B \tag{4}$$

$$R_B^A = \begin{pmatrix} c\beta & s\beta s\gamma & s\beta c\gamma \\ s\alpha s\beta & c\alpha c\gamma - s\alpha c\beta s\gamma & -c\alpha s\gamma - s\alpha c\beta c\gamma \\ c\alpha s\beta & s\alpha c\gamma + c\alpha c\beta s\gamma & -s\alpha s\gamma + c\alpha c\beta c\gamma \end{pmatrix} = \begin{pmatrix} x_l & y_l & z_l \\ x_m & y_m & z_m \\ x_n & y_n & z_n \end{pmatrix} \tag{5}$$

Employing the X-Y-Z Euler angles, where α, β, and γ denote the rotation about the X-axis, the Y-axis, and the Z-axis respectively.

Substituting the above formula yields:

$$B_i^A = \begin{bmatrix} x_p \\ y_p \\ z_p \end{bmatrix} + \begin{pmatrix} x_l & y_l & z_l \\ x_m & y_m & z_m \\ x_n & y_n & z_n \end{pmatrix} B_i^B \tag{6}$$

Where $(x_l, y_l, z_l)^T$ denotes the projection of the x-axis direction in reference frame B onto reference frame A, $(x_m, y_m, z_m)^T$ denotes the projection of the y-axis direction in reference frame B onto reference frame A, and $(x_n, y_n, z_n)^T$ denotes the projection of the z-axis direction in reference frame B onto reference frame A. B_i under $O-XYZ$ yields the absolute coordinates as follows:

$$B_1^A = \begin{bmatrix} x_p + rx_l \\ y_p + rx_m \\ z_p + rx_n \end{bmatrix} \tag{7}$$

$$B_2^A = \begin{bmatrix} x_p - \frac{1}{2}rx_l + \frac{\sqrt{3}}{2}ry_1 \\ y_p - \frac{1}{2}rx_m + \frac{\sqrt{3}}{2}ry_m \\ z_p - \frac{1}{2}rx_n + \frac{\sqrt{3}}{2}ry_n \end{bmatrix} \tag{8}$$

$$B_3^A = \begin{bmatrix} x_p - \frac{1}{2}rx_l - \frac{\sqrt{3}}{2}ry_1 \\ y_p - \frac{1}{2}rx_m - \frac{\sqrt{3}}{2}ry_m \\ z_p - \frac{1}{2}rx_n - \frac{\sqrt{3}}{2}ry_n \end{bmatrix} \tag{9}$$

Derived from the closed-loop vector relationship:

$$L_i = B_i^A - A_i^A \tag{10}$$

Derive the driving equation for the parallel mechanism, i.e., the functional relationship between the mechanism's driving input and the moving platform's positional output:

$$L_i^2 = \left|B_i^A - A_i^A\right|^2 \tag{11}$$

Separate:

$$\begin{aligned} L_1 &= \sqrt{x_p^2 + y_p^2 + z_p^2 + 2r(x_p x_l + y_p x_m + z_p x_n) + r^2 - 2R(x_p + rx_l) + R^2} \\ L_2 &= \sqrt{x_p^2 + y_p^2 + z_p^2 + R^2 + r^2 - 2Rr + r(\sqrt{3}y_p - x_p) + R(x_p - \sqrt{3}y_p)} \\ L_3 &= \sqrt{x_p^2 + y_p^2 + z_p^2 + R^2 + r^2 - 2Rr - r(\sqrt{3}y_p + x_p) + R(x_p + \sqrt{3}y_p)} \end{aligned} \tag{12}$$

For joint constraints, the parallel mechanism is subject to certain restrictions. Defining the constraint equations with respect to a fixed coordinate system and a moving coordinate system perpendicular to surface OPB_iA_i, and given that the three vectors are coplanar and linearly dependent, we obtain:

$$\begin{vmatrix} B_1^A(x) & B_1^A(y) & B_1^A(z) \\ A_1^A(x) & A_1^A(y) & A_1^A(z) \\ z_l & z_m & z_n \end{vmatrix} = \begin{vmatrix} x_p + rx_l & y_p + rx_m & z_p + rx_n \\ R & 0 & 0 \\ z_l & z_m & z_n \end{vmatrix} = 0$$

$$\begin{vmatrix} B_2^A(x) & B_2^A(y) & B_2^A(z) \\ A_2^A(x) & A_2^A(y) & A_2^A(z) \\ z_l & z_m & z_n \end{vmatrix} =$$

$$\begin{vmatrix} x_p - \frac{1}{2}rx_l + \frac{\sqrt{3}}{2}ry_1 & y_p - \frac{1}{2}rx_m + \frac{\sqrt{3}}{2}ry_m & z_p - \frac{1}{2}rx_n + \frac{\sqrt{3}}{2}ry_n \\ -\frac{1}{2}R & -\frac{\sqrt{3}}{2}R & 0 \\ z_l & z_m & z_n \end{vmatrix} = 0 \tag{13}$$

$$\begin{vmatrix} B_3^A(x) & B_3^A(y) & B_3^A(z) \\ A_3^A(x) & A_3^A(y) & A_3^A(z) \\ z_l & z_m & z_n \end{vmatrix} =$$

$$\begin{vmatrix} x_p - \frac{1}{2}rx_l - \frac{\sqrt{3}}{2}ry_1 & y_p - \frac{1}{2}rx_m - \frac{\sqrt{3}}{2}ry_m & z_p - \frac{1}{2}rx_n - \frac{\sqrt{3}}{2}ry_n \\ -\frac{1}{2}R & -\frac{\sqrt{3}}{2}R & 0 \\ z_l & z_m & z_n \end{vmatrix} = 0$$

Simplified to:

$$\begin{aligned} &\sqrt{3}(x_p - \frac{1}{2}rx_l + \frac{\sqrt{3}}{2}ry_l)z_n - (z_p - \frac{1}{2}rx_n + \frac{\sqrt{3}}{2}ry_n)z_m \\ &-\sqrt{3}(z_p - \frac{1}{2}rx_n + \frac{\sqrt{3}}{2}ry_n)z_l + (y_p - \frac{1}{2}rx_m + \frac{\sqrt{3}}{2}ry_m)z_n = 0 \\ &z_m z_p - z_n y_p = r(z_n x_m - z_m x_n) \\ &\sqrt{3}\left(x_l z_n - x_n z_l\right) + \sqrt{3}r\left(y_n z_m - y_m z_n\right) + 2\sqrt{3}(z_p z_1 - x_p z_n) \\ &+3r(y_l z_n - y_n z_l) + 2y_p z_n - 2z_p z_m + r(x_n z_m - x_m z_n) = 0 \end{aligned} \tag{14}$$

This yields three numerical analytical solutions:

$$y_p = \frac{z_m z_p - r(z_n x_m - z_m x_n)}{z_n}$$

$$x_p = \frac{2z_p z_l - r(x_n z_l - x_l z_n) - r(y_m z_n - y_n z_m)}{2z_n} \tag{15}$$

ISSN: 2688-8645

$$y_p = \frac{-3r(y_l z_n - y_n z_l) + 2z_p z_m - r(x_n z_m - x_m z_n)}{2z_n}$$

Jointly obtained:

$$x_n z_m - x_m z_n = y_n z_l - y_l z_n \tag{16}$$

In summary, the energy equation for the 3-RCU parallel mechanism's branch is as follows: Given the platform's angular displacement is small, approaching zero, the rotation matrix $\sin\phi = \phi, \cos\phi = 1$.may be approximated according to trigonometric definitions as:

$$\begin{pmatrix} x_l & y_l & z_l \\ x_m & y_m & z_m \\ x_n & y_n & z_n \end{pmatrix} = \begin{pmatrix} 1 & \beta\gamma & \beta \\ \alpha\beta & 1-\alpha\gamma & -\gamma-\alpha \\ -\beta & \alpha+\gamma & -\alpha\gamma+1 \end{pmatrix} \tag{17}$$

Substituting yields:

$$y_p \approx -2z_p\alpha, x_p = z_p\beta \tag{18}$$

Using the analytical method to determine the forward solution of motion degrees of freedom and attitude, from equation (12) we obtain:

$$L_3^2 - L_2^2 = -4\sqrt{3}(R-r)z_p\alpha \tag{19}$$

$$\frac{L_3^2 + L_2^2 - 2L_1^2}{2} = (3R+r)z_p\beta \tag{20}$$

Simplified to:

$$\alpha = \frac{L_3^2 - L_2^2}{-4\sqrt{3}(R-r)z_p}, \beta = \frac{L_3^2 + L_2^2 - 2L_1^2}{2(3R+r)z_p} \tag{21}$$

Defining $A = \frac{\beta}{\alpha}, B = z_p\beta$, substituting into (12) yields:

$$z_p = \sqrt{L_1^2 - B^2 - 4\frac{B^2}{A^2} - (R-r)^2 + 2RB - 2rB} \tag{22}$$

4. Conclusion

This paper focuses on the 3-RCU parallel robot mechanism. Composed of a moving platform, a fixed platform, and three RCU linkages, the mechanism consists of 8 components and 9 kinematic pairs. Calculations show it has three degrees of freedom: translation along the z-axis and rotation about the x and y axes. The study establishes corresponding coordinate systems and a 3D model.

It centers on kinematic analysis, adopting an analytical method to solve inverse kinematics. By deriving coordinate expressions, rotation matrices, and driving equations through structural parameters and Euler angles, and combining joint constraint conditions, analytical solutions are obtained, completing the forward solution of motion degrees of freedom and attitude.

Through accurate dynamic modeling, the research captures the nonlinear variation of support interface stiffness, providing key parameters for the subsequent design of parallel mechanisms and laying the theoretical foundation for constructing time-varying dynamic models.

Acknowledgments

Supported by College Student Innovation and Entrepreneurship Project, Design and Development of a Mirror Milling Support Machine Based on 3-RCU Parallel Mechanism, Number 202510066037.

Tianjin Science and Technology Plan Project, Development of Mechanism of Position and Attitude Change and Intelligent Control Technology for Ultra-minimally Invasive Robotic Puncture Components, 25YFYSHZ00250.

Tianjin Science and Technology Plan Project, Research and Development of Embodied Intelligent Mobile Robot Machining and Inspection System for Large and Complex Components, 24YFYSHZ00180.

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