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Research on Production Prediction of Shale Gas Wells Based on Grey Relation Analysis and Typical Decline Models

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Abstract

Against the backdrop of increasingly constrained global conventional oil and gas resources, the efficient development of shale gas, as a crucial unconventional alternative, is of paramount importance. Shale gas wells, particularly horizontal wells, exhibit a production decline behavior distinctly different from conventional gas wells, characterized by high initial rates followed by rapid decline, and ultimately transitioning to a long period of low-yield stabilization. This paper systematically investigates the production decline behavior of shale gas wells and the analytical methods of typical decline models. Firstly, it elaborates on the principles, formulas, and applicability of various models, ranging from the classical Arps decline model to modern models tailored for shale gas characteristics, such as the Duong, Power Law Exponential (PLE), Stretched Exponential Production Decline (SEPD), and Logistic Growth Model (LGM). Secondly, to address the insufficient analysis of factors influencing production decline, the Grey Relation Analysis (GRA) method is introduced. A complete analytical process—from determining analysis sequences and dimensionless processing to calculating the correlation degree—is established to quantify the impact of geological and engineering factors on the decline behavior. Finally, a comparative evaluation of the main analytical methods, including the Wattenbarger linear flow method, PLE, SEPD, Duong, and LGM, is conducted, highlighting their respective advantages, disadvantages, and application conditions. The comparison reveals differences among these models in terms of fitting accuracy, data sensitivity, flow regime identification, and early-time prediction capability. The research demonstrates that integrating grey relation analysis with appropriately selected decline models based on specific production stages and reservoir characteristics can more accurately elucidate the decline behavior of shale gas wells. This integrated approach provides a scientific basis for production forecasting, recoverable reserves estimation, and development plan optimization.

Keywords

Shale Gas Wells; Production Decline; Grey Relation Analysis.

1. Study on Production Decline Laws of Shale Gas Wells and Analysis Methods Using Typical Models

Against the backdrop of progressively declining global conventional oil and gas reserves, unconventional resources such as shale gas have emerged as critical alternative energy sources, becoming a major focus in current exploration and development [1]. Compared to conventional gas wells, shale gas horizontal wells exhibit a unique production decline profile: they typically deliver high initial production rates, which are followed by a steep decline curve. Once the production falls to a certain low level, it then tends to stabilize over the long term, with the decline rate approaching zero. Current research on production decline in shale gas wells primarily concentrates on calculating decline rates, identifying decline types, and forecasting

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production capacity. However, a systematic analysis of the key factors influencing this production decline remains relatively underdeveloped. [2]

1.1. The Arps Decline Model

Among the mathematical models for conventional production decline analysis, the Arps model is the most renowned and widely used. Proposed by American scholar J.J. Arps in 1945, the Arps decline curve was built upon the conceptual foundation of production decline established by R. Arnold and R. Anderson in 1908 [3]. The equations developed by Arps elevated the study of decline characteristics to a new, quantitative level. According to research by Professor Yu Qitai, other decline models, such as those of Kopytov, Matthews, Joshi, Gompertz, Weibull, and Logistic, can all be considered specific cases of the Arps decline model.

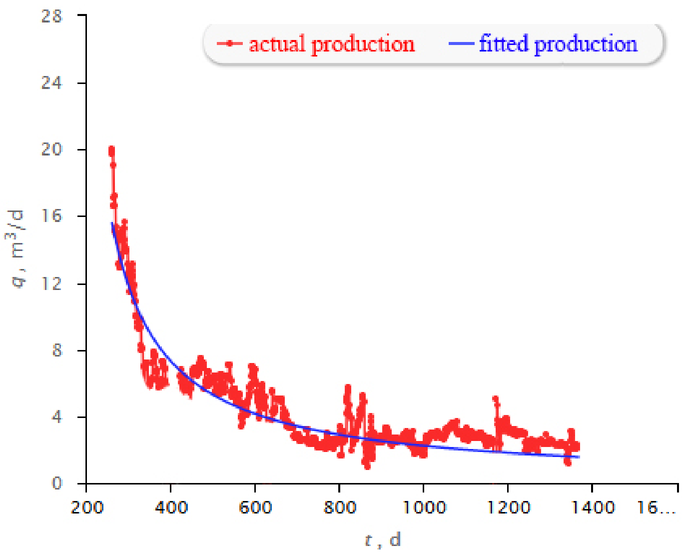

Fig 1. Arps Decline Curve Fitting

Table 1. The Three Forms of the Arps Decline Model

decreasing type	exponential decay	hyperbolic decline	harmonic decrease
decreasing exponent	n = 0	0 < n < 1	n = 1
decreasing rate	D_i	$\frac{D_i}{1+nD_it}$	$\frac{D_i}{1+D_it}$
output	$q=q_ie^{-D_it}$	$q=q_i(1+D_i\cdot nt)^{-1/n}$	$q=q_i(1+D_i\cdot t)^{-1}$
cumulative production	$N_p=\frac{q_i}{D_i}(1-e^{-D_it})$	$N_p=\frac{q_i}{D_i}\frac{1}{1-n}-\frac{q_i^n}{D_i}\frac{1}{1-n}q^{1-n}$	$N_p=\frac{q_i}{D_i}In(1+D_it)$
recoverable reserves	$N_R=\frac{E}{10000}\cdot\frac{q_i-q_0}{D_i}$	$N_R=\frac{E}{10000}\cdot\frac{q_i}{(1-n)D_i}[1-(\frac{q_0}{q_i})^{1-n}]$	$N_R=\frac{E}{10000}\cdot\frac{q_i}{D_i}In(\frac{q_i}{q_0})$

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1.2. The Fetkovich, Hsieh, and PLE Decline Models

Through an integrated study of material balance and reservoir flow models, FETKOVICH proposed a modern production decline analysis method that utilizes type curves to match production decline trends [4]. His research demonstrated that hyperbolic decline can describe the production decline process of depletion-driven gas wells under boundary-dominated flow [6]. However, this method is only applicable under constant bottom-hole pressure conditions; when the bottom-hole pressure varies, the parameters obtained through this type-curve matching become inaccurate.

Due to changes in reservoir properties during production, where the permeability to each phase continuously decreases as the gas well produces, HSIEH (2001) introduced an approach in 2001 that employs a time-dependent Darcy equation to account for these variations in reservoir properties. In this approach, the instantaneous decline rate, D, can be calculated directly from production time and rate data, while n represents the decline exponent and m denotes the time-dependent index. [5]

$$q = q_i t^{-(n+mt)} \tag{1}$$

$$D = m(1 + Lnt) + n/t \tag{2}$$

In 2008, ILK et al. proposed the Power-Law Exponential Decline Model (PLE model) to address the issue of unrealistically high production forecasts associated with the continuously decreasing decline rate in the late-time period of hyperbolic decline. In this model, D_i represents the decline rate at a unit time, and D_∞ is the decline rate as the well's drainage transitions to the boundary-dominated flow regime within the Stimulated Reservoir Volume (SRV). If early-time production data contains significant noise, the estimation of D_i can be substantially erroneous, often necessitating a trial-and-error procedure to determine the model parameters.

$$q = q_i \exp\left(-D_\infty t - D_n t^n\right) \tag{3}$$

Where $D_n = D_1/n$.

2. A Study on the Correlation of Factors Affecting Shale Gas Well Production Decline Based on Grey Relational Analysis

Grey Relational Analysis (GRA) is a technique that measures the correlation between factors based on the similarity of their developmental trends. This method quantifies the relationship by comparing the geometric similarity between the reference and comparative sequences: the closer the curves align, the higher the grey relational grade, and vice versa [8]. Applying GRA to shale gas well production analysis helps effectively identify the relative influence of various geological and engineering factors on production decline, thereby providing a critical basis for optimizing production strategies and enhancing recovery.

2.1. Determination of Analysis Sequences

The first step involves defining the reference sequence that characterizes the system's behavior and the comparative sequences that influence it [13]. Let Y_0' be the reference sequence and Y_i' (where $i = 1,2,\dots,m$) be the comparative sequences. These $m + 1$ data series form the following matrix:

$$(Y`_0, Y`_i, Y`_m) = \begin{bmatrix} y`_0(1) & y`_1(1) & \cdots & y`_m(1) \\ y`_0(2) & y`_1(2) & \cdots & y`_m(2) \\ \vdots & \vdots & \cdots & \vdots \\ y`_0(n) & y`_1(n) & \cdots & y`_m(n) \end{bmatrix}_{n(m+1)} \tag{4}$$

In the Grey Relational Analysis model, Y_0' is defined as the reference sequence while $Y_i' = \{y_i'(1), y_i'(2), \cdots, y_i'(n)\}^T$ (where $i = 1,2,\dots,m$) represents the comparative sequences, with m denoting the total number of comparative sequences and n specifying the length of each data sequence. [11]

Due to the distinct physical meanings of various factors within a system, the dimensions of data may vary, which can hinder comparability or lead to inaccurate conclusions during analysis. Therefore, when conducting grey relational analysis, it is generally necessary to perform dimensionless data processing. Commonly used dimensionless methods include the initial value method, the mean value method, and the range normalization method, among others.

2.2. Calculation of the Difference Sequence, Maximum Difference, and Minimum Difference

The absolute differences between the reference sequence and the comparative sequences are determined, forming the following absolute difference matrix:

$$\begin{bmatrix} \Delta_{01}(1) & \Delta_{02}(1) & \cdots & \Delta_{0m}(1) \\ \Delta_{01}(2) & \Delta_{02}(2) & \cdots & \Delta_{0m}(2) \\ \vdots & \vdots & \cdots & \vdots \\ \Delta_{01}(n) & \Delta_{02}(n) & \cdots & \Delta_{0m}(n) \end{bmatrix}_{n\times m} \tag{5}$$

$$\Delta_{0i}(k) = |y_0(k) - y_i(k)|$$

The maximum and minimum values in the absolute difference matrix (5) are the global maximum difference and the global minimum difference, respectively:

$$\max\{\Delta_{0i}(k)\} = \Delta(max)$$

$$\min\{\Delta_{0i}(k)\} = \Delta(max)$$

The data in the absolute difference matrix (5) are transformed as follows:

$$\lambda_{0i}(k) = \frac{\Delta(min) + \rho\Delta(max)}{\Delta_{0i}(k) + \rho\Delta(max)}$$

The grey relational coefficient matrix is given as follows:

$$\begin{bmatrix} \lambda_{01}(1) & \lambda_{02}(1) & \cdots & \lambda_{0m}(1) \\ \lambda_{01}(2) & \lambda_{02}(2) & \cdots & \lambda_{0m}(2) \\ \vdots & \vdots & \cdots & \vdots \\ \lambda_{01}(n) & \lambda_{02}(n) & \cdots & \lambda_{0m}(n) \end{bmatrix}_{n\times m} \tag{6}$$

In this formula, $\rho \in (0, \infty)$ is denoted as the distinguishing coefficient. A smaller value of ρ dictates a greater distinguishing power. While its value is typically set within the interval (0, 1), the specific choice can be determined case by case. The optimal distinguishing effect is achieved when $\rho \leq 0.5463$, and a value of $\rho = 0.5$ is commonly adopted. The grey relational coefficient $\lambda_{0i}(k)$ is a positive number not exceeding 1. A smaller absolute difference $\Delta_{0i}(k)$ results in a larger coefficient $\lambda_{0i}(k)$, which reflects the degree of correlation between the comparative sequence Y_i' and the reference sequence Y_0' at the k-th period. [15]

2.3. Computation and Ranking of Relational Grades

As the grey relational coefficients represent the degree of correlation between the comparative and reference sequences at each time point (i.e., each point on the curve), they are multiple and dispersed. This dispersion makes it inconvenient for an overall comparison [17]. Therefore, it is necessary to aggregate these coefficients from various moments into a single value. The average of these coefficients is taken as a quantitative measure of the overall correlation between the sequences, known as the relational grade r_i, which is calculated as follows:

$$r_i = \frac{1}{n}\sum_{k=1}^{n} \lambda_{0i}(k) \tag{7}$$

The relational degrees of the m comparative sequences relative to the same reference sequence are arranged in descending order, forming a relational order [16]. This order reflects the relative superiority or inferiority of each comparative sequence in relation to the reference sequence.

3. Comparative Analysis and Application Assessment of Production Decline Methods for Shale Gas Wells

3.1. The Wattenbarger Linear Flow Method

Given the extremely low permeability of shale, transient linear flow of fluids within shale gas reservoirs persists for a considerable duration. Consequently, analytical methods can be employed to estimate the original gas in place (OGIP). Ibrahim and Wattenbarger proposed an equation for estimating reservoir pore volume based on the end time of transient flow [19]. The early-form derivation of this equation yields the following formula for calculating OGIP:

$$G = f_{cp}\frac{200.8T}{\emptyset(\mu_g C_t)_i}\cdot\frac{\sqrt{t_{elf}}}{m_{CPL}}\left(\frac{\emptyset S_{gi}}{B_{gi}} + V_L\frac{P_i}{P_i+P_L}\right) \tag{8}$$

$$m_{CPL} = \frac{3.154T}{h\sqrt{(\emptyset\mu_g C_t)_i}}\cdot\frac{1}{P_{pi}-P_{pwf}}\cdot\frac{1}{x_f\sqrt{K}} \tag{9}$$

$$f_{cp} = 1 - 0.0852p_D - 0.0857p_D^2 \tag{10}$$

$$p_D = \frac{\varphi_i - \varphi_{wf}}{\varphi_i} \tag{11}$$

In the formulation, the key variables are defined as follows: G is the Original Gas in Place (OGIP) in m^3, calculated based on total reservoir compressibility (C_t, MPa^{-1}), the initial gas formation volume factor (B_{gi}), and initial gas saturation (S_{gi}). The pressure conditions are characterized by the initial reservoir pressure (p_i, MPa), Langmuir pressure (p_L, MPa), and Langmuir volume ($V_L\ m^3$) for adsorption description. The analysis incorporates a correction factor for pressure depletion-induced slope error (f_{cp}) and the dimensionless pressure drop (p_D). Flow behavior is described using pseudo-pressures at initial and flowing bottom-hole conditions (ϕ_i, ϕ_{wf}, MPa), the slope of the transient linear flow region (m_{CPL}), and the duration of this flow regime (t_{elf}, d). Reservoir properties include absolute temperature (T, K), effective thickness (h, m), hydraulic fracture half-length (x_f, m), permeability (K, mD), porosity (ϕ, %), and gas viscosity (μ_g, mPa·s). The pseudo-pressures at initial and bottom-hole conditions (p_{pi}, p_{pwf}, MPa) are also utilized. The subscript i denotes initial reservoir conditions.

Equation (8) is exclusively applicable to producing wells during the linear flow regime, whose duration is given by:

$$t_{elf} = \left(\frac{y_e\sqrt{(\emptyset\mu_g C_t)_i}}{2\times 0.159\sqrt{K}}\right)^2 \tag{12}$$

where y_e is the distance of flow to the boundary (m).

The Wattenbarger linear flow method is advantageous as it: (1) accurately estimates recoverable reserves by modeling long-term transient flow; (2) facilitates reservoir flow regime identification; and (3) incorporates Langmuir parameters (p_L, V_L) to compute reserves from both adsorbed and free gas in various well types. Its principal limitation is the sensitivity to shale permeability, a parameter with high inherent uncertainty in ultra-low permeability formations [20]. Variability in permeability directly affects the calculated linear flow duration and, consequently, the reserve estimates.

3.2. The Stretched Exponential Production Decline (SEPD) Model

Valkó and Lee proposed a new methodology for decline analysis, analogous to the PLE model, termed the Stretched Exponential Production Decline (SEPD) model. This model was developed through a statistical study of production data from multiple shale gas wells. In the SEPD model, the relationship between production rate and time is described by the following equation:

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$$q_g(t) = q_i \exp\left[-(\frac{t}{\tau})^n\right] \tag{13}$$

where τ is a coefficient to be determined by fitting historical production data. A key distinction between the SEPD and PLE models lies in their treatment of late-time decline behavior. The SEPD model does not account for the late-time decline characteristic, denoted as D_∞ in the PLE model; in the SEPD model, D_∞ is invariably zero. Consequently, the parameter τ in the SEPD model is equivalent to $1/\widehat{D}_i^{1/n}$ in the PLE model.

The Stretched Exponential Production Decline (SEPD) model offers several distinct advantages for unconventional gas reservoir evaluation. Firstly, as a model specifically designed for estimating the Estimated Ultimate Recovery (EUR) of unconventional resources, it typically provides more reliable results than conventional Arps decline analysis. Secondly, it can accurately characterize the production decline behavior of tight shale gas wells. Thirdly, it facilitates the identification of different flow regimes, effectively distinguishing between transient flow and boundary-dominated flow. Finally, by providing an analytical formula for cumulative gas production over time, it enables a more straightforward and smoother fit to production data with discrete trends.

However, the model is subject to certain limitations. It demonstrates high sensitivity to the selection and quality of individual data points. Furthermore, its capability for early-stage prediction of the maximum recoverable volume is limited; reliable forecasts are generally only attainable after the well reaches boundary-dominated flow, often resulting in overall conservative EUR estimates. Moreover, the process of fitting its parameters is iterative and time-consuming, typically requiring the development and use of specialized software for practical implementation.

3.3. The Duong Decline Model

Based on the long-term linear flow production characteristics observed in shale gas wells, Duong proposed a model for forecasting production and estimating recoverable reserves, subsequently named the Duong decline model after its originator. For a shale gas well that exhibits a sustained fracture linear flow or bilinear flow regime throughout its production life, the gas production rate can be characterized by a power-law function of time:

$$q_g(t) = q_i t^{-n} \tag{14}$$

In Equation (15), the parameter n takes a value of 0.5, indicative of linear flow, or 0.25, indicative of bilinear flow. To enhance the model's flexibility for matching production data, Duong introduced a variable parameter m, replacing the fixed exponent of time t. The modified formulation is presented as follows:

$$q_g(t) = q_i t^{-m} e^{\frac{a}{1-m}(t^{1-m}-1)} \tag{15}$$

where a and m are fitting parameters in the model.

The Duong decline model offers several distinct advantages for shale gas reservoir evaluation. Primarily, it provides a framework for forecasting production and estimating recoverable reserves, typically delivering superior results compared to conventional Arps decline analysis. Furthermore, the model effectively matches production data during the transient flow regime of shale gas wells. Additionally, it enables early-time prediction of both production rates and ultimate recovery.

However, the model is subject to several notable limitations. It demonstrates high sensitivity to individual data points, necessitating pre-processing steps such as differential smoothing or averaging of raw production data prior to analysis. Moreover, the model's characteristic curve does not directly exhibit the signature of late-time boundary-dominated flow. Consequently, the base Duong model requires modification to accurately match production data once pseudo-steady state flow is established. Finally, the process of fitting its parameters is iterative and

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time-consuming, generally requiring the use of specialized software for practical implementation.

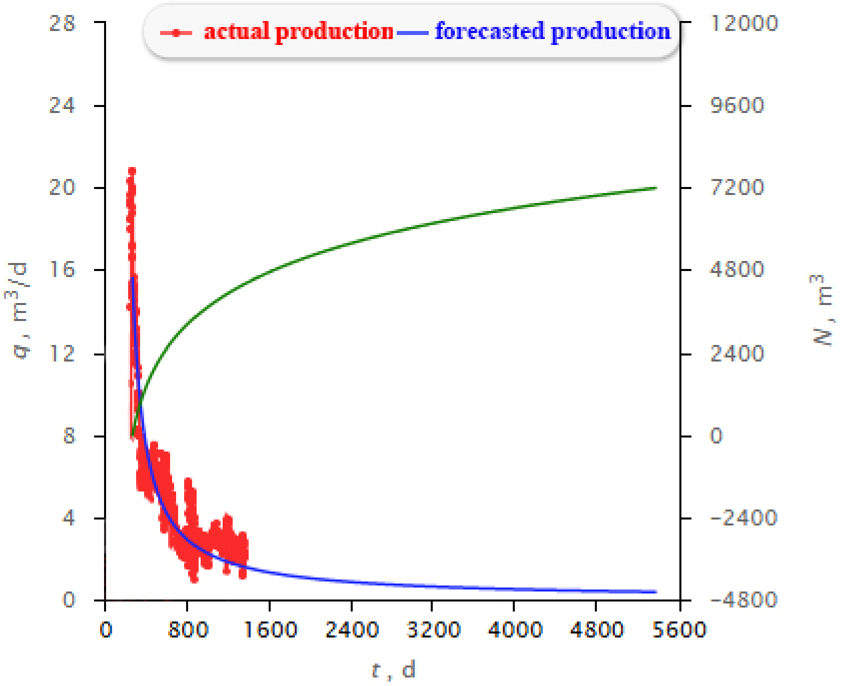

Fig 2. Arps Decline Production Forecast

3.4. The Logistic Growth Model (LGM)

Clark et al. proposed the Logistic Growth Model (LGM), which is applicable to single-well production forecasting:

$$q_g(t) = \frac{Q_f n \alpha t^{n-1}}{(\alpha+t^n)^2} \tag{16}$$

where Q_f is the ultimate predicted recoverable volume per well without economic constraints (in 10^8 m^3/d), and α is the time (in months) required to recover 50% of this ultimate volume.

The Logistic Growth Model (LGM) presents several advantages for shale gas well performance analysis [7]. Firstly, as a model designed for forecasting production and estimating recoverable reserves, it typically yields more accurate results than the conventional Arps decline model. Secondly, it demonstrates a robust capability in matching production data from both the transient flow and boundary-dominated flow regimes. Thirdly, the model effectively aids in distinguishing between different flow states throughout the well's production life.

However, the application of the LGM is subject to certain limitations. A primary constraint is its high sensitivity to individual data points, which often necessitates pre-processing steps such as differential smoothing or averaging of raw production data before analysis. Furthermore, the model's output is strongly governed by the pre-defined ultimate recoverable volume (Q_f); when this value is uncertain, the model predictions can exhibit significant uncertainty and non-unique solutions [10]. Finally, the process of calibrating the model parameters is iterative and time-consuming, generally requiring the support of specialized software for efficient implementation.

4. Summary

This article systematically establishes an integrated methodology for analyzing the production performance of shale gas wells. Research indicates that the production decline behavior of shale gas wells is significantly different from that of conventional gas reservoirs, characterized by unique production features: high initial rates followed by rapid decline, ultimately transitioning to long-term low-yield stabilization. Given this complexity, a single decline model is insufficient for achieving accurate predictions throughout the entire production lifecycle.

This study conducts a comparative analysis of typical models—including the Wattenbarger linear flow method, SEPD, Duong, PLE, and LGM—revealing their distinct characteristics in terms of fitting accuracy, data sensitivity, and flow regime identification capability. Furthermore, it innovatively introduces the Grey Relational Analysis (GRA) method, establishing a complete analytical process from determining analysis sequences to calculating relational grades, which effectively quantifies the impact of geological and engineering factors on production decline.

The research ultimately confirms that combining grey relational analysis for key factor identification with the appropriate selection of decline models based on specific production stages and reservoir characteristics can more accurately elucidate the decline behavior of shale gas wells. This research approach—"factor analysis, model optimization, integrated prediction"—not only provides a more scientific analytical method for production forecasting and recoverable reserves estimation of shale gas wells but also establishes a theoretical foundation for optimizing development plans and enhancing stimulation strategies. It holds significant practical guidance for promoting the efficient development of shale gas resources.

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