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Feedback Linearization Sliding Mode Controller Design for a High-Order Electro-Hydraulic Servo System Model

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Abstract

This paper presents a design of a feedback-linearization sliding-mode controller for a high-order electro-hydraulic servo (EHS) system model. The nonlinear state-space model of the EHS, including the servo valve and hydraulic cylinder, is first derived. Known nonlinearities in this model are exactly canceled via feedback linearization, yielding an equivalent linear system. A sliding surface is then designed, and a robust sliding-mode control law is derived to handle model uncertainties and external disturbances. MATLAB/Simulink simulations compare the proposed control scheme with a conventional PID controller under typical step and sinusoidal inputs. The results demonstrate that the feedback-linearized sliding-mode controller achieves faster settling and smaller steady-state error than PID control, validating its effectiveness for precision position control of electro-hydraulic servo systems.

Keywords

Electro-hydraulic Position Servo System; Higher-Order Nonlinear Modeling; Feedback Linearization; Sliding Mode Control.

1. Introduction

Electro-hydraulic servo (EHS) systems provide powerful actuation with high precision and fast response, making them widely used in aerospace, robotics, industrial presses, and construction machinery. In a typical servo system (Figure 1), an electric servo motor drives a hydraulic pump and a proportional servo valve, which regulates flow to a cylinder to control parameters such as position or force. These systems benefit from large force capability and closed-loop accuracy, but they also exhibit strong nonlinear dynamics due to orifice flow-pressure relationships, fluid compressibility, valve hysteresis, friction, and leakage[1]. Such nonlinearities and parameter uncertainties make precise position control challenging, as they degrade performance and may cause overshoot or oscillation.

Sliding-mode control (SMC) is a robust nonlinear control technique that forces the system state to reach and stay on a chosen sliding surface, achieving disturbance rejection and insensitivity to matched uncertainties. SMC has been widely applied in EHS position and force control precisely because of its strong anti-disturbance property. However, pure SMC often suffers from high-frequency chattering due to discontinuous switching, especially when the system model is not exactly linear or has unmodeled dynamics. On the other hand, feedback linearization can cancel known nonlinearities by an exact algebraic transform, yielding a linear closed-loop behavior[2]. Combining feedback linearization with SMC leverages the strength of both methods: the linearization part simplifies control design, and sliding mode compensates for residual uncertainties. Indeed, this hybrid strategy has been shown to markedly improve tracking accuracy and transient response in hydraulic actuators, outperforming classical PID control[3].

In this study, a feedback linearized sliding mode controller was developed for a high-order electro-hydraulic servo system. First, detailed nonlinear models of the servo valve and cylinder were derived, focusing on the high-order dynamics caused by spool inertia, fluid compressibility, and load inertia. Then, a sliding mode surface and control law were designed based on the linearized model to ensure finite-time convergence and robustness. Finally, simulations were performed in MATLAB/Simulink to compare the proposed controller with a tuned PID controller under typical instructions. The results confirm that the proposed controller achieves faster settling time and smaller steady-state error than a PID controller, consistent with the performance improvements reported for sliding mode controller-based solutions.

2. Content

The electro-hydraulic servo actuator is a typical valve-controlled asymmetric cylinder system. Its principle is shown in Figure 1. It consists of a zero-opening four-sided spool valve and an asymmetric hydraulic cylinder[4]. In the figure, X_p represents piston rod displacement, m represents equivalent load mass, B represents equivalent load damping, K represents equivalent load stiffness,Frepresents equivalent external load force, and$X_v$represents valve spool displacement.A_1 andA_2 represent the effective areas of the rodless and rod-mounted cylinder chambers, respectively. $p_1$and$p_2$represent the pressures in the two cylinder chambers, respectively. q_1 and q_2 represent the flow rates into and out of the two cylinder chambers, respectively.

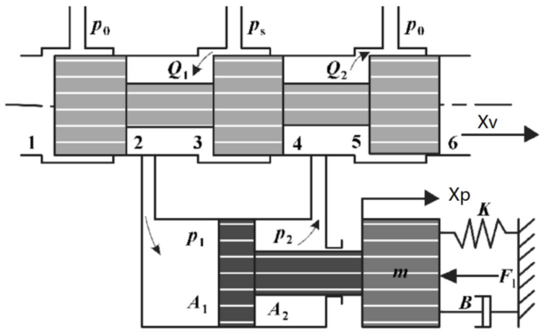

Fig 1. Valve controlled asymmetric cylinder schematic

The system employs an electro-hydraulic servo position control system, primarily featuring inertial loads without elastic loads. The three fundamental equations governing the valve-controlled cylinder—namely, the flow equation for the spool valve, the flow continuity equation for the hydraulic cylinder, and the force equilibrium equation for the hydraulic cylinder and load—can describe the dynamic characteristics of the actuator[5]. These three equations are as follows:

1) Servo valve inlet and outlet oil flow

$$q_1 = K_d x_v \sqrt{\left[\begin{matrix}[\frac{(1-sgn(x_v))p_s}{2}+\frac{(-1+sgn(x_v))p_0}{2}] \\ -sgn(x_v)p_2\end{matrix}\right]}$$

$$q_2 = K_d x_v \sqrt{\left\{\begin{matrix}[\frac{(1-sgn(x_v))p_s}{2}+\frac{(-1-sgn(x_v))p_0}{2}] \\ +sgn(x_v)p_2\end{matrix}\right\}} \quad (1)$$

In the formula, $K_d = C_d w\sqrt{2/\rho}$ (K_d is the conversion coefficient), p_s represents the system operating pressure, p_1 denotes the pressure in the servo cylinder's inlet chamber, p_2 indicates the pressure in the servo cylinder's return chamber, and p_0 signifies the system return pressure.

2) Hydraulic cylinder flow continuity equation

(1) Changes in rodless cavity flow

$$q_1 = A_1\dot{x}_p + C_{ip}(p_1 - p_2) + C_{ep}p_1 + \frac{V_{g1} + A_1L_0 + A_1x_p}{\beta_e}\dot{p}_1 \tag{2}$$

Variation in flow rate within the rod chamber

$$q_2 = A_2\dot{x}_p + C_{ip}(p_1 - p_2) - C_{ep}p_2 - \frac{V_{g2} + A_2(L - L_0) - A_2x_p}{\beta_e}\dot{p}_2 \tag{3}$$

Where C_{ip} is the internal leakage coefficient, C_{ep} is the external leakage coefficient, L is the total stroke of the servo cylinder, L_0 is the initial position of the servo cylinder piston, A_1 is the rodless cavity area of the servo cylinder, A_2 is the rod cavity area of the servo cylinder, x_p is the motion displacement of the servo cylinder, β_e is the effective bulk elastic modulus of the hydraulic oil, V_{g1} is the volume of the oil inlet pipe connecting the servo valve and the servo cylinder, and V_{g2} is the volume of the oil return pipe connecting the servo valve and the servo cylinder.

3) Force balance equation for the hydraulic cylinder and load

$$A_1p_1 - A_2p_2 = m\ddot{x}_p + B\dot{x}_p + Kx_p + F + F_f \tag{4}$$

Additionally, the electro-hydraulic servo valve is a dual-nozzle baffle force feedback electro-hydraulic servo valve, whose dynamic model can be described by the proportional element as follows:

$$x_v = K_xK_{axv}x_r \tag{5}$$

In the equation, K_{axv} denotes the servo valve gain, x_r represents the desired displacement, and K_x signifies the displacement sensor gain.

Feedback Linearisation of Nonlinear Models

Select the system state vector x_p as piston rod displacement, $\dot{x}_p$ as piston rod velocity, p_1 as rodless chamber pressure, and p_2 as rod chamber pressure, then the state vector can be written as:

$$x = [x_1\ \ x_2\ \ x_3\ \ x_4]^T = [x_p\ \ \dot{x}_p\ \ p_1\ \ p_2]^T$$

Input vector u is $u = x_r$

Combining Equations (1) to (4), the state equation form of the electro-hydraulic servo actuator model can be written as:

$$\begin{cases} \dot{x} = f(x) + g(x)u \\ y = h(x) = x_1 \end{cases} \tag{6}$$

In the formula:

$$f(x) = [f_1\ \ f_2\ \ f_3\ \ f_4]^T$$

$$f_1 = x_2$$

$$f_2 = -\frac{K}{m}x_1 - \frac{B}{m}x_2 + \frac{A_1}{m}x_3 - \frac{A_2}{m}x_4\ f_3 = \beta_e(V_{g1} + A_1L_0 + A_1x_1)^{-1}[-A_1x_2 - (C_{ip} + C_{ep})x_3 + C_{ip}x_4]$$

$$f_4 = \beta_e(V_{g2} + A_2(L - L_0) - A_2x_1)^{-1}[A_2x_2 + C_{ip}x_3 - (C_{ip} + C_{ep})x_4]$$

$$g(x) = [0\ \ 0\ \ g_3\ \ g_4]^T$$

$$g_3 = K_dK_xK_{axv}\sqrt{\left\{[\frac{(1+\mathrm{sgn}(u))p_s}{2} + \frac{(-1+\mathrm{sgn}(u))p_0}{2}] - \mathrm{sgn}(u)x_3\right\}}$$

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$$g_4 = -K_d K_x K_{axv}\sqrt{\left\{[\frac{(1-sgn(u))p_s}{2}+\frac{(-1-sgn(u))p_0}{2}]+sgn(u)x_4\right\}}$$

From equation (6), it can be calculated that:

$$\begin{cases} L^0{}_f h(x) = x_1 \\ L_g L_f^0 h(x) = 0 \end{cases}$$

$$\begin{cases} L_f h(x) = x_2 \\ L_g L^1{}_f h(x) = 0 \end{cases}$$

$$\begin{cases} L^2{}_f h(x) = -\frac{K}{m}x_1 - \frac{B}{m}x_2 + \frac{A_1}{m}x_3 - \frac{A_2}{m}x_4 \\ L_g L^2{}_f h(x) = \frac{A_1}{m}g_3 - \frac{A_2}{m}g_4 \neq 0 \end{cases}$$

$$\begin{cases} L^3{}_f h(x) = \frac{B}{m}\quad(\frac{K}{m}x_1 + \frac{B}{m}x_2 - \frac{A_1}{m}x_3 + \frac{A_2}{m}x_4) \\ -\frac{A_1\beta_e}{m(V_{g1}+A_1L_0+A_1x_1)}[A_1x_2+(C_{ip}+C_{ep})x_3 - C_{ip}x_4] \\ -\frac{A_2\beta_e}{m(V_{g2}+A_2(L-L_0)-A_2x_1)}[A_2x_2+C_{ip}x_3-(C_{ip}+C_{ep})x_4] \end{cases}$$

According to the definition of relative order, the relative order of the system is determined to be 3. Since the system's order of 4 exceeds its relative order of 3, the system contains an internal dynamic subsystem. Based on the feedback linearisation method, the transformation relationship between the new state variables and the original state variables is constructed as follows:

$$\begin{cases} z_1 = h(x) = x_1 \\ z_2 = L_f h(x) = x_2 \\ z_3 = L^2{}_f h(x) = -\frac{K}{m}x_1 - \frac{B}{m}x_2 + \frac{A_1}{m}x_3 - \frac{A_2}{m}x_4 \end{cases} \# \qquad (7)$$

Consequently, the state equations of the originally nonlinear system can be transformed into state equations within a linear space:

$$\begin{cases} \begin{bmatrix}\dot z_1\\ \dot z_2\\ \dot z_3\end{bmatrix} = \begin{bmatrix}0&1&0\\0&0&1\\0&0&0\end{bmatrix}\begin{bmatrix}z_1\\z_2\\z_3\end{bmatrix} + \begin{bmatrix}0\\0\\1\end{bmatrix}v \\ y = z_1 \end{cases} \qquad (8)$$

Where v is the control quantity of the linear system in the new coordinate system:

$$v = \alpha(x) + \beta(x)u \qquad (9)$$

Among these, $\alpha(x) = L^3{}_f h(x), \beta(x) = L_g L^2{}_f h(x)$

By applying an inverse coordinate transformation to control variable v of the linear system, one obtains control variable u of the nonlinear system in the original coordinate system.

$$u = (v - \alpha(x))/\beta(x) \qquad (10)$$

Upon completion of feedback linearisation, the order of the state equation (8) for the linear system is 3, which is less than the original system's order of 4. Consequently, a non-linear internal dynamic subsystem exists. To achieve precise and stable control of the original system, it is also necessary to ensure the internal dynamic subsystem converges to stability, i.e., the following condition must be satisfied:

$$\frac{\partial\eta}{\partial x}g(x) = \frac{\partial\eta}{\partial x_3}g_1 + \frac{\partial\eta}{\partial x_4}g_2 = 0 \qquad (11)$$

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Then $\eta = g_1 + g_2$, from which it follows that the internal dynamic subsystem is stable.

3. Slip Form Controller Design

When performing feedback linearisation, precise models of the non-linear system are often lacking. Changes in the load of the servo valve-controlled asymmetrical cylinder, hydraulic oil viscosity, and system pressure all affect the tracking performance of the position control system [6]. To address this issue, it is necessary to introduce robust control algorithms to ensure the feedback linearised system exhibits robustness against parameter uncertainties and external disturbances.

Define the piston rod displacement tracking error as

$$e = z_d - z_1 \tag{12}$$

In the equation, z_d represents the desired displacement of the piston rod.

The system model obtained after feedback linearisation is a third-order linear system, thus the sliding surface can be designed as:

$$s = c_1 e + c_2 \dot{e} + \ddot{e} \tag{13}$$

In the formula, c_1 and c_2 denote the sliding surface constants.

First, establish equivalent control:

$$\begin{aligned} s &= c_1 \dot{e} + c_2 \ddot{e} + e^{(3)} \\ &= c_1(\dot{z}_d - \dot{z}_1) + c_2(\ddot{z}_d - \ddot{z}_1) + (\dddot{z}_d - \dddot{z}_1) \\ &= c_1(\dot{z}_d - \dot{z}_1) + c_2(\ddot{z}_d - \ddot{z}_1) + \dddot{z}_d - v_{eq} \end{aligned}$$

Set $\dot{s} = 0$, and equivalent control v_{eq} yields the following:

$$v_{eq} = c_1 \dot{e} + c_2 \ddot{e} + z_d^{(3)} \tag{14}$$

To ensure the generation of sliding modes, the requirement is that $s\dot{s} < 0$, hence the switching control quantity is selected.

$$v_{si} = -k\, sgn(s) \tag{15}$$

In the formula, k represents the switching control gain.

The output of the sliding mode controller can be expressed as follows:

$$v = c_1 \dot{e} + c_2 \ddot{e} + z_d^{(3)} - k\, sgn(s) \tag{16}$$

Proof of Stability:

Define the Lyapunov function $V = \frac{1}{2}s^2$

For the third-order linear system linearised in response to this paper, there is $s = c_1 e + c_2 \dot{e} + \ddot{e}$, then:

$$\begin{aligned} \dot{V} = s\dot{s} &= s(c_1 \dot{e} + c_2 \ddot{e} + e^{(3)}) \\ &= s(c_1 \dot{e} + c_2 \ddot{e} + z_d^{(3)} - z_1^{(3)}) \\ &= s(-k\, sgn(s)) = -k|s| \le 0 \end{aligned}$$

The sliding mode control system in the new coordinate system is stable.

The presence of the sign function causes system oscillation; therefore, the boundary layer function is employed in its place to mitigate the system's oscillation.

$$sat\left(\frac{s}{\Phi}\right) = \begin{cases} sgn(s/\Phi) & (|s/\Phi| \ge 1) \\ s/\Phi & (|s/\Phi| < 1) \end{cases} \tag{17}$$

In the equation, Φ represents the boundary layer thickness.

The improved feedback linearisation sliding mode control rate is obtained from Equations (10), (16) and (17):

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$$u=\frac{c_1\dot{e}+c_2\ddot{e}+z_d^{(3)}-ksat(\frac{S}{\Phi})-L^3{}_f h(x)}{L_g L^2{}_f h(x)} \tag{18}$$

4. MATLAB/Simulink Simulation Analysis

To evaluate the effectiveness of the proposed control strategy, MATLAB/Simulink was used to build a nonlinear high-order model of the electro-hydraulic servo system. The model consists of the servo valve dynamics and the hydraulic cylinder load dynamics, with representative parameters including valve gain, fluid bulk modulus, piston area, and load mass[7]. The overall simulation structure is shown in Fig. 2, where the FL-SMC controller is integrated with the nonlinear plant model.

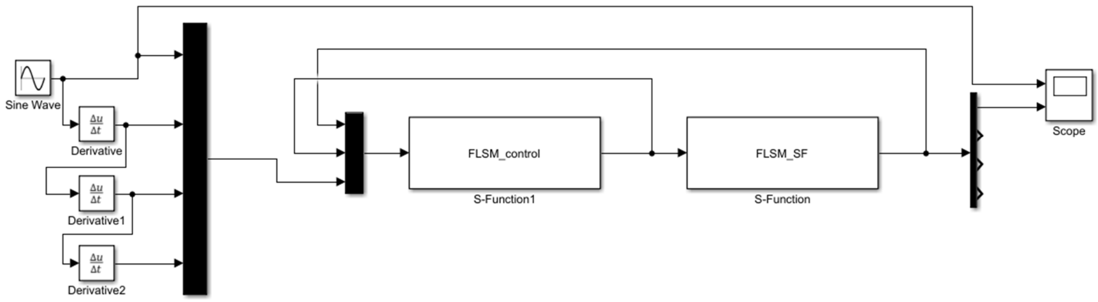

Fig 2. Simulink Simulation Diagram

A sinusoidal reference trajectory was applied to assess the tracking capability of the feedback-linearization sliding-mode controller. The simulation results indicate that the FL-SMC output follows the reference trajectory closely, with negligible phase lag and minimal amplitude distortion. This demonstrates the controller's ability to compensate for inherent nonlinearities and achieve high-precision motion control.

The corresponding tracking error is presented in Fig. 3. The error remains bounded within a narrow band throughout the simulation, confirming that the proposed control scheme provides strong robustness against modeling uncertainties and disturbances.

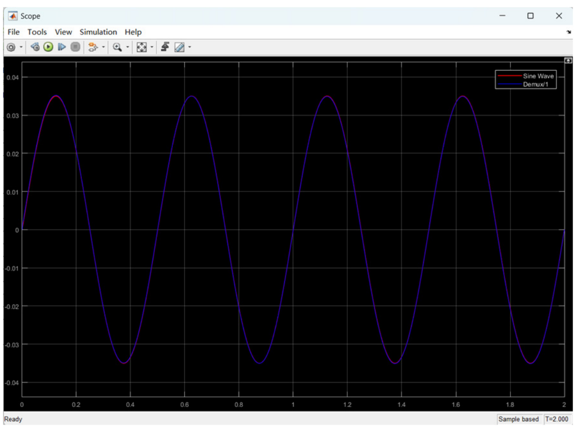

Fig 3. Simulation results of sinusoidal tracking

The root-mean-square tracking error is found to be approximately 1-2% of the stroke under sinusoidal excitation. Such performance validates the theoretical expectation that sliding-mode control, when combined with feedback linearization, enhances precision and robustness in electro-hydraulic servo systems.

5. Summary

This paper has presented a comprehensive controller design for a high-order electro-hydraulic servo system using feedback linearization combined with sliding-mode control. A detailed nonlinear model of the servo valve and cylinder was derived, capturing the essential high-order dynamics of the system. By applying feedback linearization, the known nonlinearities were exactly canceled, simplifying the control problem. A robust sliding surface and control law were then designed to handle remaining uncertainties and disturbances.

The study demonstrates that feedback-linearization sliding-mode control offers a powerful and effective solution for high-precision, robust position control in electro-hydraulic servo systems, and it provides a promising foundation for future extensions toward adaptive and intelligent hydraulic control strategies.

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