

# Suppression of Seismic Random Noise Using CEEMDAN Combined with EWT

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## Abstract

During the process of seismic data acquisition, random noise is generated by the natural environment, electronic noise from acquisition instruments, and coupling issues. These noises are superimposed on the effective seismic signals, reducing the signal-to-noise ratio of the data and causing difficulties for subsequent processing and interpretation. Therefore, eliminating the interference of random noise is a particularly important step. In this regard, this paper proposes Complementary Ensemble Empirical Mode Decomposition with Adaptive Noise (CEEMDAN) combined Empirical Wavelet Transform (EWT) suppresses random noise in earthquakes. Through the simulation of the processing procedure and the processing of actual data, it is concluded that the method proposed in this paper has a significant effect on suppressing random noise.

## Keywords

CEEMDAN; EWT; Random Noise Suppression; IMF.

## 1. Introduction

Empirical Mode Decomposition (EMD), proposed by Huang in 1998, is a method for analyzing signal time and frequency. Its principle is to perform EMD decomposition on nonlinear and non-stationary signals, obtain Intrinsic Mode Functions (IMF) components, and then reconstruct the IMF [1]. Under normal circumstances, the high-frequency seismic signals contain more noise, and the effective signals are concentrated on the low-frequency signals. Therefore, EMD is highly effective in processing nonlinear and non-stationary signals. However, EMD also has problems such as modal aliasing and endpoint effects. Huang et al. [2] proposed Ensemble Empirical Mode Decomposition (EEMD) based on EMD. This method adds auxiliary white noise to the original signal before decomposing the signal, making the distribution of extremum points more balanced and solving the problem of modal aliasing. However, due to the introduction of additional noise by EEMD, the residual noise during reconstruction is large, resulting in a large amount of computation and low computational efficiency. Therefore, in 2010, Yeh et al. proposed Complete Ensemble Empirical Mode Decomposition (CEEMD) based on EEMD, improving the addition of auxiliary white noise in the EEMD method to the addition of positive and negative paired white noise [3]. In 2011, Torres et al. improved on the research of EMD, EEMD and CEEMD and proposed the Adaptive Noise Complete Set Empirical Mode Decomposition (CEEMDAN) algorithm, further optimizing the noise adaptive mechanism. This method first adaptively decomposes the auxiliary white noise [4]. Then, it was added to the corresponding IMF components. Compared with EMD, CEEMDAN basically did not show any modal aliasing phenomenon. Compared with EEMD, the reconstruction error of CEEMDAN was almost zero. Compared with CEEMD, the computational efficiency of CEEMDAN was significantly improved. In 2015, Gomez et al. proposed a new method that can accelerate the computational speed of adaptive noise complete Set Empirical Mode Decomposition

(CEEMDAN), replacing interpolation calculation with convolution, effectively improving the denoising efficiency of seismic data, and showing obvious advantages in the processing of microseismic and seismic exploration reflection waves [5].

Empirical Wavelet Transform (EWT) is a non-stationary signal processing method proposed by Giles in 2013 [6]. It integrates the adaptive decomposition concept of the EMD method and the tightly supported characteristic framework of wavelet transform theory, providing a brand-new adaptive time-frequency analysis idea for signal processing. Compared with the wavelet transform, the empirical wavelet transform also has corresponding scale functions and wavelet basis functions. However, unlike the wavelet transform, its scale functions and wavelet basis functions are not fixed but dynamically generated based on the local characteristics of the data, which endows the empirical wavelet transform with more flexible analytical capabilities. Compared with EMD, EWT can adaptively select frequency bands, overcoming the modal aliasing problem caused by the discontinuity of signal time-frequency scales. Meanwhile, it has a complete and reliable mathematical theoretical basis, low computational complexity, and can also overcome the over-enveloping and under-enveloping problems in the EMD method. Therefore, it is widely applied in the field of signal processing.

The idea of this paper is to first use CEEMDAN to separate the original signal, so that some of the IMF components contain a large amount of noise and some contain effective signals. Then, EWT is used to process each IMF component. Finally, the processed IMF components are reconstructed to obtain the denoising signal.

## 2. Method and Principle

### 2.1. Principle of the CEEMDAN Method

First, add white noise of different amplitudes to the original signal. Perform EMD decomposition on these signals with added white noise to obtain the first IMF component. After averaging these IMF components, obtain the final first IMF component. Then, add white noise to calculate the second IMF component until the decomposition is completed. The calculation formula is as follows:

Adding noise  $n_i(t)$  with an amplitude of  $\varepsilon$  to the original signal  $x(t)$  yields the disturbed signal  $x_i(t)$ .

$$x_i(t) = x(t) + \varepsilon n_i(t) \quad (1)$$

Among them,  $i=1,2,\dots,M$ , where  $M$  represents the total average number of times.

Perform EMD decomposition on the disturbance signal  $x_i(t)$  to obtain the first set of IMF:

$$x_i(t) = \sum_{j=1}^M c_{ij}(t) + r_i(t) \quad (2)$$

Among them,  $c_{ij}(t)$  is the  $j$ -th IMF of the  $i$ -th noisy signal, and  $r_i(t)$  is the remainder.

Averaging the IMF and remainder of all disturbance signals yields:

$$c_j(t) = \frac{1}{M} \sum_{i=1}^M c_{ij}(t) \quad (3)$$

$$r(t) = \frac{1}{M} \sum_{i=1}^M r_i(t) \quad (4)$$

After the above steps, the effective signal is decomposed into one IMF component and one remainder:

$$x(t) = c_1(t) + r_1(t) \quad (5)$$

Finally, take the remaining term  $r_1(t)$  as the original signal and repeat the above calculation steps until  $r_i(t)$  becomes a monotonic function or is sufficiently small. The result obtained is the final result of the CEEMDAN decomposition.

## 2.2. Principles of the EWT Method

First, normalize the spectrum of the signal, then divide the intervals, and on this basis, construct the empirical wavelet according to the construction method of Meyer wavelet. The scale function  $\varphi(t)$  is defined as:

$$\hat{\varphi}_n(\omega) = 1, |\omega| \leq \omega_n - \tau_n \quad (6)$$

$$\hat{\varphi}_n(\omega) = \cos \left[ \frac{\pi}{2} \beta \left( \frac{1}{2\tau_n} (|\omega| - \omega_n + \tau_n) \right) \right], \omega_n - \tau_n \leq |\omega| \leq \omega_n - \tau_n \quad (7)$$

$$\hat{\varphi}_n(\omega) = 0, \text{ else} \quad (8)$$

The empirical wavelet function  $\psi(t)$  is defined as:

$$\hat{\psi}_n(\omega) = 1, \omega_n + \tau_n \leq |\omega| \leq \omega_{n+1} - \tau_{n+1} \quad (9)$$

$$\hat{\psi}_n(\omega) = \cos \left[ \frac{\pi}{2} \beta \left( \frac{1}{2\tau_{n+1}} (|\omega| - \omega_{n+1} + \tau_{n+1}) \right) \right], \omega_{n+1} - \tau_{n+1} \leq |\omega| \leq \omega_{n+1} + \tau_{n+1} \quad (10)$$

$$\hat{\psi}_n(\omega) = \sin \left[ \frac{\pi}{2} \beta \left( \frac{1}{2\tau_n} (|\omega| - \omega_{n+1} + \tau_{n+1}) \right) \right], \omega_n - \tau_n \leq |\omega| \leq \omega_{n+1} + \tau_{n+1} \quad (11)$$

$$\hat{\psi}_n(\omega) = 0, \text{ else} \quad (12)$$

The common definitions of the transition function  $\beta(x)$ , the transition section  $\tau_n$  and the coefficient  $\gamma$  are as follows:

$$\beta(x) = x^4(36 - 84x + 70x^2 - 20x^3) \quad (13)$$

$$\tau_n = \gamma \omega_n \quad (14)$$

$$\gamma < \min \left( \frac{\omega_{n+1} - \omega_n}{\omega_{n+1} + \omega_n} \right) \quad (15)$$

Similar to the wavelet transform, the detail coefficient and approximation coefficient of EWT are also obtained by the inner product of the scale function  $\varphi(t)$  and the empirical wavelet function  $\psi(t)$  with the original signal:

$$W_f^\varepsilon(n, t) = \langle f(t), \varphi_n(t) \rangle = \int f(\tau) \overline{\varphi_n(\tau - t)} d\tau = F^{-1}(f(\omega) \hat{\varphi}_n(\omega)) \quad (16)$$

$$W_f^\varepsilon(0, t) = \langle f(t), \varphi_n(t) \rangle = \int f(\tau) \overline{\varphi_n(\tau - t)} d\tau = F^{-1}(f(\omega) \hat{\varphi}_n(\omega)) \quad (17)$$

Thus, the empirical mode component can be expressed as:

$$f_0(t) = W_f^\varepsilon(0, t) * \varphi_1(t) \quad (18)$$

$$f_n(t) = W_f^\varepsilon(n, t) * \varphi_n(t) \quad (19)$$

## 2.3. Joint Denoising Principle

First, the original signal is decomposed using CEEMDAN. Then, the IMF components are screened. Finally, the screened components are reconstructed. In this paper, the screening method for IMF components is the Cross-Correlation Function (CCF), and the cross-correlation

function analysis is the correlation between two sequences under different time lags. The value of the cross-correlation function is a specific numerical measure of the correlation between two time series at different lag orders  $k$ . Its value range is between  $[-1,1]$ , indicating the degree of linear correlation between two sequences. The formula for calculating the cross-correlation function is as follows:

$$CCF(k) = \frac{\sum_{t=1}^{n-k} (X_t - \bar{X})(Y_{t+k} - \bar{Y})}{\sqrt{\sum_{t=1}^n (X_t - \bar{X})^2 \sum_{t=1}^n (Y_t - \bar{Y})^2}} \quad (20)$$

Among them,  $X_t$  and  $Y_t$  are two time series, and  $k$  is the lag order, which can be positive, negative or zero. When  $k > 0$ ,  $Y_t$  lags behind  $X_t$ ; when  $k < 0$ ,  $X_t$  lags behind  $Y_t$ ; when  $k = 0$ , there is no lag.

The formula for calculating the value of the cross-correlation function is as follows:

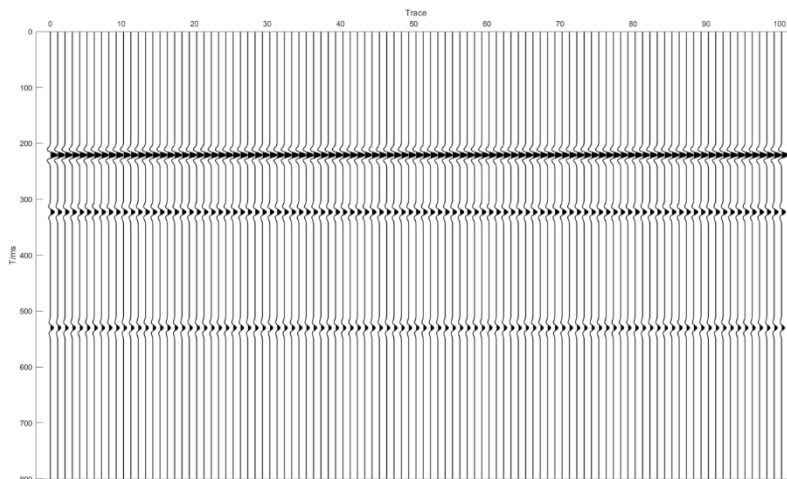
$$CCF(k) = \frac{Cov(X_t, Y_{t+k})}{\sqrt{Var(X_t) \cdot Var(Y_t)}} \quad (21)$$

Here,  $Cov(X_t, Y_{t+k})$  is the covariance of  $X_t$  and  $Y_{t+k}$ , and  $Var(X_t)$  and  $Var(Y_t)$  are the variances of  $X_t$  and  $Y_t$ , respectively.

The range of the cross-correlation function value is  $[-1,1]$ . When the cross-correlation function value is 1, it indicates a complete positive correlation; when it is 0, it indicates no correlation; and when it is -1, it indicates a complete negative correlation.

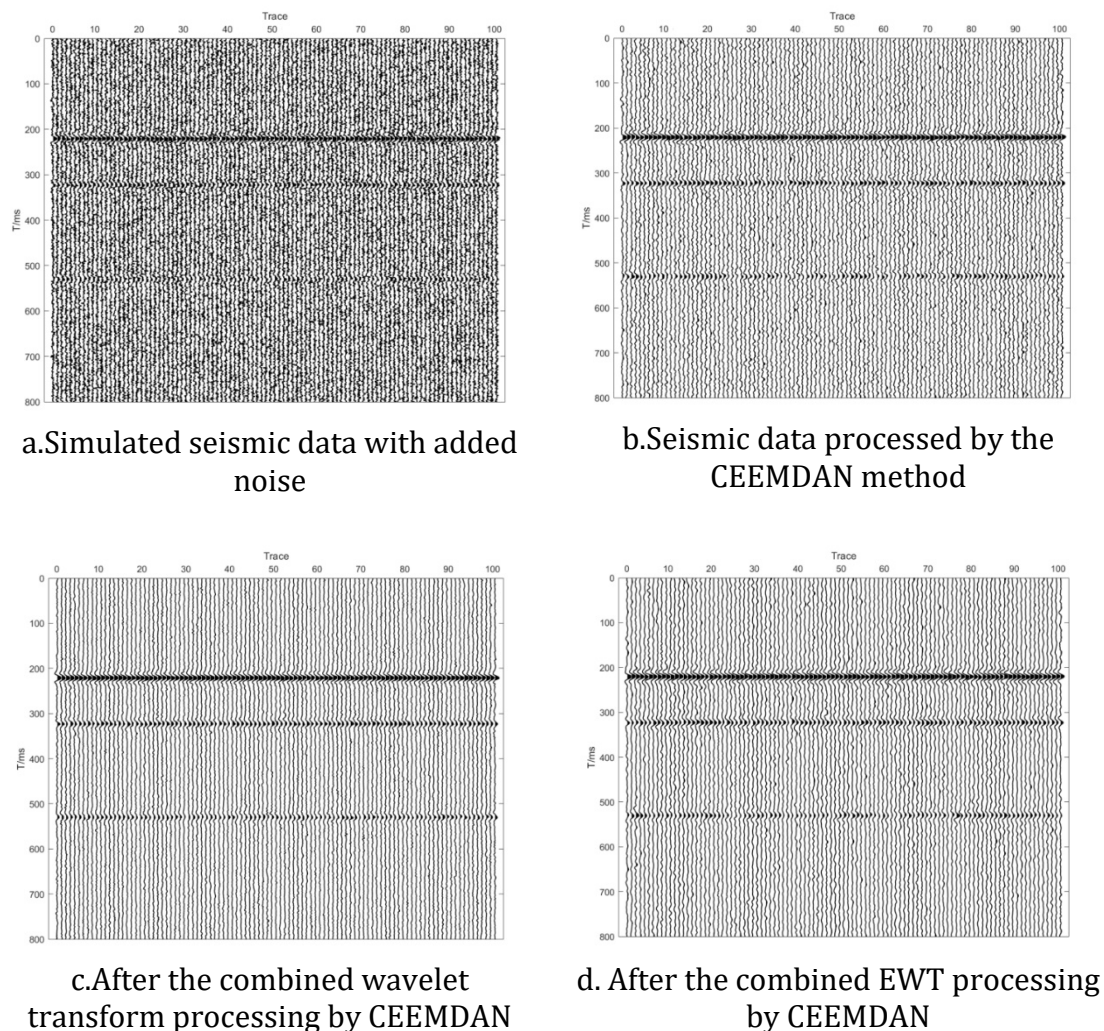
### 3. Simulation Data Analysis

First, simulate multiple seismic records as shown in Figure 1, which consists of four strata with a total of three reflection interfaces. The first layer is the surface soil layer with a stratum velocity of 500m/s, a stratum density of 1.2g/cm<sup>3</sup>, and a stratum thickness of 300m. The second layer is a clay layer with a formation velocity of 900m/s, a formation density of 2g/cm<sup>3</sup>, and a formation thickness of 1000m. The third layer is a sandstone layer with a formation velocity of 1400m/s, a formation density of 2.3g/cm<sup>3</sup>, and a formation thickness of 3000m. The fourth layer is a limestone layer with a formation velocity of 2000m/s and a formation density of 2.5g/cm<sup>3</sup>. Then random noise was added, as shown in Figure 2a. After adding noise, the signal-to-noise ratio of the simulated seismic record was 1.94dB. It can be seen that random noise seriously affects the quality of seismic records.



**Figure 1.** Simulated seismic data

Various denoising processes were carried out on the seismic records after noise addition, including separate CEEMDAN processing, CEEMDAN combined with wavelet transform processing, and CEEMDAN combined with EWT processing for comparison. As shown in Figure 2, it can be seen from the figure that compared with the noisy simulated seismic records, the records processed by CEEMDAN have some noise suppressed, and the clarity of the in-phase axis has been improved. However, there are still some residual noises, which result in the continuity and clarity of the signal not being ideal. CEEMDAN combined with wavelet threshold denoising further enhances the noise suppression capability. Noise is removed more effectively, the main features of the signal are well preserved, the in-phase axis is clear and continuous, and a better balance is achieved between denoising and signal fidelity. The noise suppression effect of the records after denoising by the CEEMDAN combined with the EWT method is the best. The noise is almost completely suppressed, and the details and characteristics of the signal are retained to the greatest extent. The in-phase axis is the clearest and most accurate. It is the best method among the three for noise suppression of the simulated seismic records with added noise.



**Figure 2.** Comparison of simulated earthquake data before and after processing

According to the data analysis recorded in Table 1, the CEEMDAN combined with EWT method not only has a higher signal-to-noise ratio than the CEEMDAN combined with wavelet threshold denoising, but also has a higher operational efficiency. Therefore, overall, the denoising effect of the CEEMDAN combined with EWT is relatively the best.

**Table 1.** Comparison of processing effects

|                               | SNR/dB | RMSE   | Time/s     |
|-------------------------------|--------|--------|------------|
| CEEMDAN                       | 4.16   | 0.1101 | 150.081399 |
| CEEMDAN combined with wavelet | 7.96   | 0.1178 | 300.717515 |
| CEEMDAN combined with EWT     | 9.73   | 0.1253 | 189.086633 |

#### 4. Analysis of Actual Data

Figure 3a shows the single-shot record of the original seismic data. It can be seen that the waveform is rather chaotic, the in-phase axis is not clear and has poor continuity, and there is a large amount of random noise. Among them, there are several high-frequency random noises that develop extremely obviously under the influence of geological factors. The noise suppression effects are shown in Figures 3b and 3c respectively after CEEMDAN combined wavelet transform processing and CEEMDAN combined EWT processing. It can be seen that the noise of the seismic record after CEEMDAN combined wavelet transform processing is significantly reduced, but there is a disconnection phenomenon in the non-phase axis at 1700ms. The seismic records processed by CEEMDAN and EWT not only have a clear and continuous phase axis from 1400ms to 1600ms, but also have a significantly restored phase axis at 1700ms compared to the results of CEEMDAN and EWT combined wavelet transform processing.

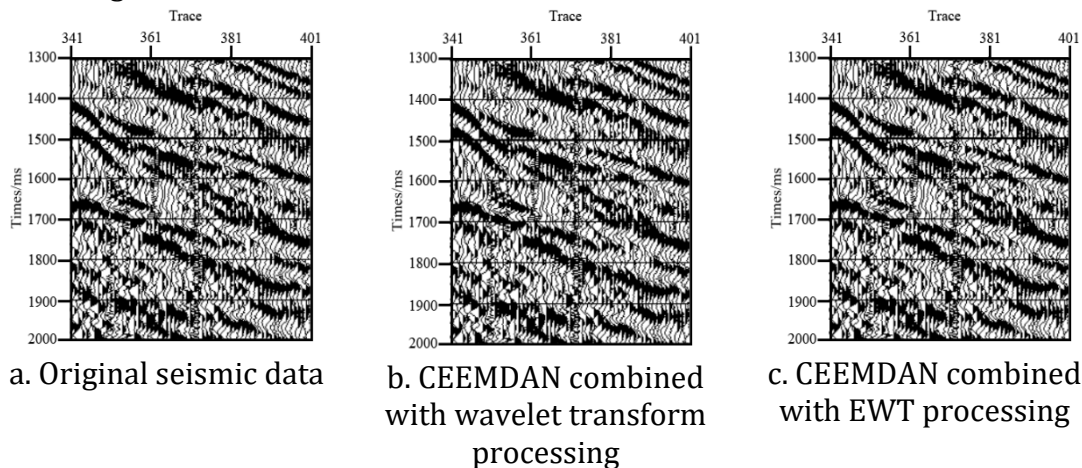
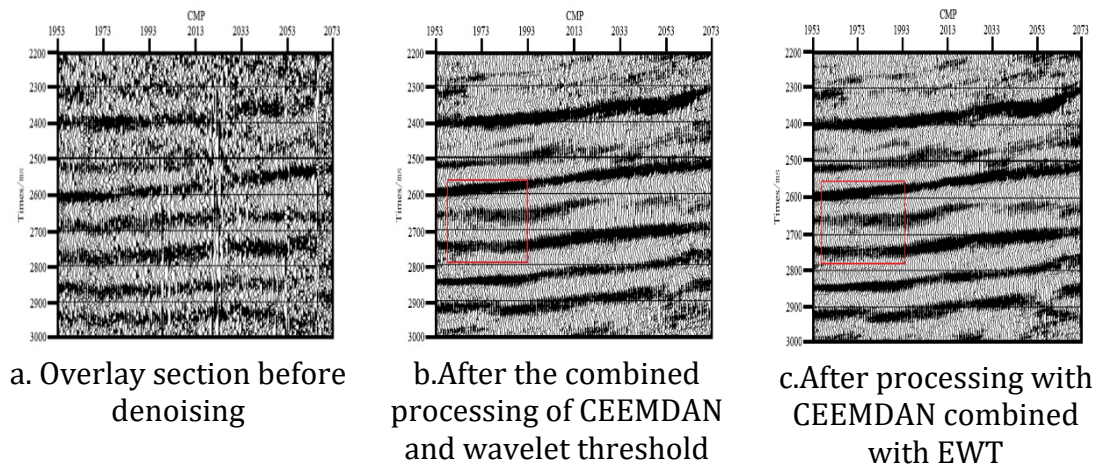
**Figure 3.** Comparison of single shot record before and after denoising**Figure 4.** Overlay section before and after denoising by different methods

Figure 4 shows the comparison of the superimposed profiles before and after noise suppression by different methods. Figure 4c is the superimposed profile after noise suppression by the CEEMDAN combined with EWT method. The overall effect is clearer than that of the CEEMDAN combined with wavelet threshold denoising processing. For example, the waveforms within the range of CMP1953 to CMP1973 have significantly reduced interference. The clarity of the in-phase axis from 2600ms to 2700ms is clearer than the result of CEEMDAN combined with wavelet threshold denoising, and the superimposed profile waveform processed by CEEMDAN combined with EWT method is smoother.

## 5. Conclusion

(1) As an efficient time-frequency analysis method, EMD's characteristic of decomposing signals has a good effect on noise suppression of seismic signals. On this basis, the improved algorithm CEEMDAN has more advantages in the effect and stability of random noise suppression.

(2) Using CEEMDAN alone to suppress seismic random noise will lead to incomplete suppression of random noise and incomplete retention of effective signals. On this basis, combined with EWT to further process the decomposed IMF components, the effect of random noise suppression is further improved.

(3) The application of CEEMDAN combined with EWT denoising and CEEMDAN combined with wavelet threshold denoising in actual seismic data processing was compared. When processing single-shot records, CEEMDAN combined with EWT has obvious advantages over CEEMDAN combined with wavelet threshold, and the effect of superimposing profiles after noise suppression is also better. It can be regarded as an effective processing method for suppressing random noise in seismic data.

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