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Parameter Sensitivity Analysis of the Exponential Filter Model

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Abstract

This study employs the ERA5-Land soil moisture dataset and applies an exponential filter to estimate root-zone soil moisture across the Indochina Peninsula. The filtered estimates are then compared against the original root-zone soil moisture provided by ERA5-Land. The focus is on examining the parameter range of the exponential filter model—that is, how closely the estimated root-zone soil moisture matches the known ERA5-Land values under different filter time constants. The objective is to assess the impact of filter parameter selection on the estimation results and to provide guidance for inverting root-zone soil moisture from surface observations in remote sensing applications. This approach offers an effective solution in complex terrains where in situ validation data are scarce.

Keywords

Soil Moisture; Exponential Filter; Root-zone Soil Moisture Inversion.

1. Introduction

Soil moisture is a critical component of terrestrial ecosystems, directly influencing vegetation growth, ecosystem services, and the global hydrological cycle. Consequently, soil moisture has become a key parameter of interest across agriculture, meteorology, ecology, and hydrology[1]. However, obtaining soil moisture data at multiple scales and with high spatiotemporal resolution remains a major challenge for improving the fidelity of land-surface and atmospheric models.

Conventional in-situ measurements of soil moisture offer high accuracy but are time-consuming, labor-intensive, and insufficiently dense to support large-scale, continuous monitoring. With the rapid advancement of satellite remote sensing, it has become feasible to observe soil moisture over broad regions in near real-time. Optical sensors typically infer soil moisture indirectly by exploiting relationships between moisture and vegetation indices or land-surface temperature—methods such as the Vegetation Drought Index or thermal inertia approaches[2]. Yet optical retrievals are highly susceptible to interference from clouds, precipitation, aerosols, and variations in solar irradiance, particularly in persistently cloudy or rainy regions, resulting in poor temporal continuity, limited spatial coverage, and compromised accuracy.

In contrast, passive microwave remote sensing is far less affected by atmospheric conditions and offers wide spatial coverage, short revisit intervals, and a high sensitivity to soil moisture changes. As a result, it has emerged as the most widely adopted and effective approach for satellite-based soil moisture monitoring[3]. Recent technological progress has further shifted microwave observations from high-frequency, single-angle measurements toward lower-frequency, multi-angle configurations that are better suited for soil moisture retrieval.

Despite these advances in surface soil moisture estimation, inferring root-zone soil moisture remains a significant challenge. The exponential filter model—an idealized, two-layer framework that characterizes vertical moisture transfer with a single time-constant parameter T—has been widely used to simulate root-zone moisture dynamics. However, traditional calibration of T relies heavily on ground-based observations, limiting model applicability in

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regions where such data are scarce. Given that the ERA5-Land reanalysis provides multi-layer soil moisture profiles, one can apply the exponential filter to the ERA5-Land surface moisture to infer root-zone moisture, then compare these estimates against the native ERA5-Land root-zone data. This approach offers a novel pathway for determining appropriate T ranges and thereby enhances root-zone soil moisture estimation in data-limited, complex terrains.

2. Methods and Data

2.1. Study Area

The Indochina Peninsula features highly complex terrain spanning tropical and subtropical zones, with landforms that include mountains, plateaus, and plains, and with extensive coverage of forests and agricultural fields. Influenced by the tropical monsoon climate, the region's precipitation exhibits pronounced seasonal and interannual variability: dry and rain-scarce winters contrasted by summer rainfall that is highly dependent on monsoon strength. Monsoon anomalies often trigger rainfall deficits, resulting in frequent seasonal and interannual droughts. Agriculture, a pillar of the peninsula's economy, is extremely sensitive to water availability. Root-zone soil moisture directly governs water uptake by plant roots and thus crop growth, making it a critical indicator of drought development. The complexity of the terrain limits the feasibility of large-scale in-situ monitoring; consequently, passive microwave remote sensing of surface soil moisture coupled with an exponential filter model to infer root-zone moisture has become widely adopted. Since the accuracy of this inversion hinges on the proper choice of the filter's time-constant parameter T, reliably estimating T is essential for improving root-zone moisture retrievals.

2.2. ERA5-Land Dataset

ERA5-Land is the fifth-generation global land-surface reanalysis produced by the European Centre for Medium-Range Weather Forecasts (ECMWF). It offers high spatiotemporal resolution and a broad suite of variables, making it a cornerstone dataset for meteorological, hydrological, and environmental studies. By assimilating multi-source observations—including satellite retrievals, ground stations, and radiosondes—within an advanced numerical modeling framework, ERA5-Land delivers hourly data from 1979 to the present at a spatial resolution of 0.1°×0.1°. The dataset provides both surface (0–7 cm) and root-zone (28–100 cm) soil moisture fields, which we leverage to validate and calibrate the exponential filter's parameter T by comparing filter-derived estimates against the native ERA5-Land root-zone values.

2.3. Study

The exponential filter[4] is a widely used signal processing technique, primarily employed to smooth time-series data or eliminate noise. Its core concept involves weighted averaging, where more recent observations are given higher weights, while earlier observations are down-weighted exponentially. By applying the exponential filter, time-series data can be smoothed to remove high-frequency noise and extract meaningful patterns.

The exponential filter model is based on a simplified two-layer water balance scheme, in which the only considered flux between the surface and the underlying storage layer is infiltration. This approach assumes that the flux from the surface to the subsurface is proportional to the difference in soil moisture content between the two layers. The recursive form of the exponential filter is given as:

$$RSM(t_n) = \frac{\sum_i^n sm(t_i) e^{-\frac{t_n - t_i}{T}}}{\sum_i^n e^{-\frac{t_n - t_i}{T}}} \tag{1}$$

where RSM represents the root-zone soil moisture, SM denotes the surface soil moisture, and t_n denotes the timestamp (in days).

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3. Results

To evaluate the influence of the exponential filter parameter T on the smoothing performance of soil moisture, this study used surface soil moisture (0–7 cm) from the ERA5-Land dataset as the input time series and root-zone soil moisture (28–100 cm) as the reference target. The root-zone soil moisture estimates were obtained using the exponential filter formula with weighted averaging. Specifically, we tested a range of T values—25, 30, 35, 40, 45, 50, 55, and 60 days—to compare and analyze the smoothing effect under different parameter settings.

Subsequently, results from both dry and humid regions were examined to assess how the optimal T value varies under different soil moisture conditions. The results are illustrated in the following figures:

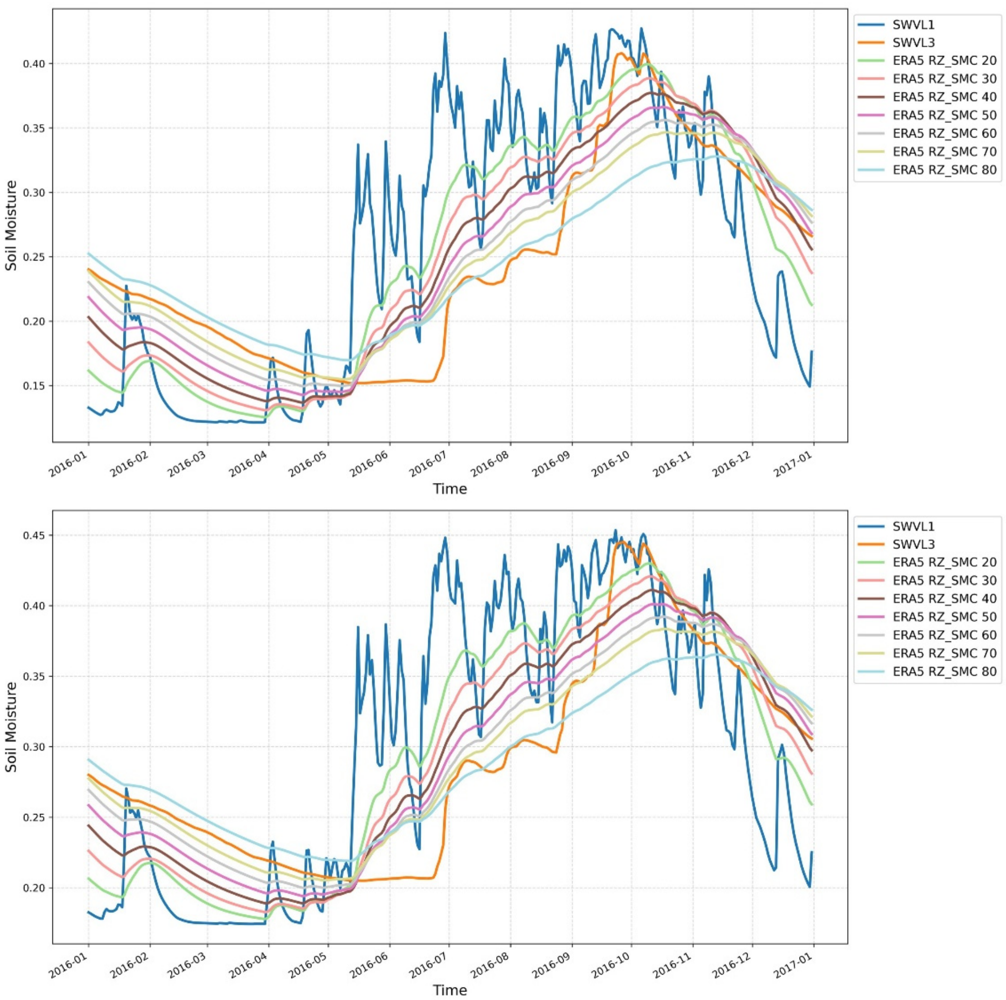

Figure 1. Filtering results of different T-values in humid areas

In this study, SWVL1 represents surface soil moisture, SWVL3 denotes root-zone soil moisture, and "ERA5 RZ_SMC 20", "ERA5 RZ_SMC 30", etc., refer to the results derived from the exponential filter using parameter values T = 20, T = 30, respectively. To assess the regional applicability of the T parameter, typical humid and arid regions were selected, and the filter results under different T values were compared against ERA5-Land root-zone soil moisture.

Figure 1 presents the parameter sensitivity experiment results for the humid region. The figure shows that when T < 60, the filtered results exhibit clear surface moisture characteristics, indicating that smaller time constants tend to capture the rapid fluctuations of surface soil moisture. As T increases, the filter output achieves the best fit with root-zone soil moisture in the range of T = 60–70, effectively capturing the delayed response of moisture in the root zone. However, when T exceeds 80, over-smoothing becomes evident, leading to a loss of seasonal variability and distortion of deep soil moisture dynamics.

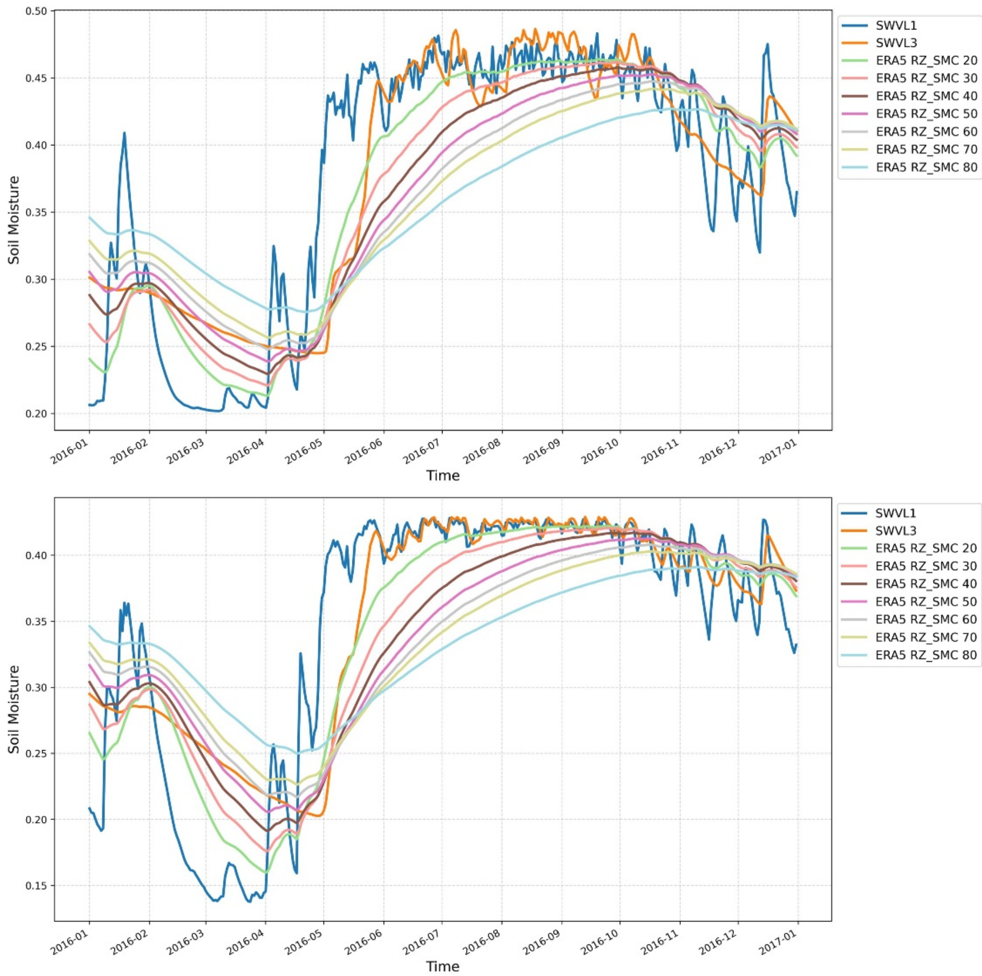

Figure 2. Filter results of different T values in the dry area

Figure 2 illustrates the results for the arid region under different T values. In contrast to the humid region, when soil moisture is low, T = 50 yields the best fit between the filtered results and ERA5-Land root-zone soil moisture. When soil moisture is higher, smaller T values result in better approximation, but a strong convergence between surface and root-zone soil moisture is observed, significantly reducing the sensitivity of the T parameter. This indicates that during high-moisture periods in arid areas, the vertical gradient of soil moisture diminishes, fundamentally limiting the applicability of the conventional exponential filter. Therefore, it is recommended to incorporate physically based models adjusted by soil texture parameters for improved estimation.

4. Conclusion

Based on ERA5-Land reanalysis data, this study estimated root-zone soil moisture using surface soil moisture and employed the original ERA5-Land root-zone data as a benchmark. The optimal selection of the exponential filter parameter T was investigated in two representative regions: arid and humid. The main conclusions are as follows:

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In humid regions, when T falls within the range of 60–70, the exponential filter effectively balances the surface response and the delayed dynamics of the root zone, resulting in the best fit with the ERA5-Land root-zone soil moisture.In arid regions, under low soil moisture conditions, the optimal T is approximately 50. However, during periods of relatively high moisture, the surface and root-zone moisture levels tend to converge, significantly reducing the sensitivity of the T parameter. This suggests that the conventional exponential filter becomes less effective in distinguishing vertical moisture gradients under such conditions.

In summary, the parameter T in the exponential filter model should be dynamically selected based on regional moisture conditions. For humid regions, a moderately high T value (60–70) is recommended to balance rapid response and lag detection. In arid regions, soil moisture status must be considered, and it is advisable to incorporate correction models based on physical parameters such as soil texture to improve the accuracy and generalizability of root-zone soil moisture simulations.

References

[1] CROW W T, KOSTER R D, REICHLE R H, et al. Relevance of time-varying and time-invariant retrieval error sources on the utility of spaceborne soil moisture products [J] Geophysical Research Letters, 2005, 32(24).

[2] STISEN S, SANDHOLT I, NøRGAARD A, et al. Combining the triangle method with thermal inertia to estimate regional evapotranspiration--Applied to MSG-SEVIRI data in the Senegal River basin [J] Remote sensing of Environment, 2008, 112(3): 1242-55.

[3] JIA Y, LI Z. Progress in land surface temperature retrieval from passive microwave remotely sensed data [J] Prog Geogr, 2006, 25(3): 96-105.

[4] ALBERGEL C, RüDIGER C, PELLARIN T, et al. From near-surface to root-zone soil moisture using an exponential filter: an assessment of the method based on in-situ observations and model simulations [J] Hydrology and Earth System Sciences, 2008, 12 (6): 1323-37.